وزارة التربية والتعليم العالي المديريّة العامة للتربية دائرة الامتحانـات الرسميّة	امتحانات الشهادة المتوسطة	دورة العام ٢٠٢٢ الاستثنائية السبت ٣ ايلول ٢٠٢٢
عدد المسائل: خمس	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الإلتزام بترتيب المسائل الواردة في المسابقة.

I- (3 points)

Given the two numbers $A = \sqrt{28} - 2\sqrt{3} - \sqrt{7} + \sqrt{27}$ and $B = \dfrac{4}{\sqrt{7}+\sqrt{3}}$.

Show all the steps of calculation.

1) Show that $A = \sqrt{7} + \sqrt{3}$ and $B = \sqrt{7} - \sqrt{3}$.
2) Verify that $A \times B$ is a natural number.
3) Calculate m so that the table below is a table of proportionality.

A	m
10	B

II- (4 points)

Given: $A(x) = (x+9)^2 - 3(x+1)(x+9)$ and $B(x) = x^2 + 10x + 9$.

1) Show that $A(x) = -2(x-3)(x+9)$.
2) Verify that $B(x) = (x+1)(x+9)$.
3) Solve the equation $A(x) = B(x)$.
4) Given: $F(x) = \dfrac{-2(x-3)(x+9)}{(x+1)(x+9)}$.
   a. For what values of $x, F(x)$ is defined ?
   b. Simplify $F(x)$.
   c. Does the equation $F(x) = -2$ have a solution? Justify.

III- (2 points)

Walid draws 24 geometric figures of circles and squares.

The number of squares represents 25% of these figures.

1) Determine the number of squares and that of circles.
2) Walid colored 11 figures in blue where 5 of them are squares.

   He colored the other figures in red.

   Show that $\dfrac{2}{3}$ of circles are colored in red.

IV- (5.5 points)

In the plane referred to an orthonormal system $(x'Ox; y'Oy)$, consider the points $A(6\ ;\ 2)$, $B(-2\ ;\ 2)$ and $C(2\ ;\ 0)$.

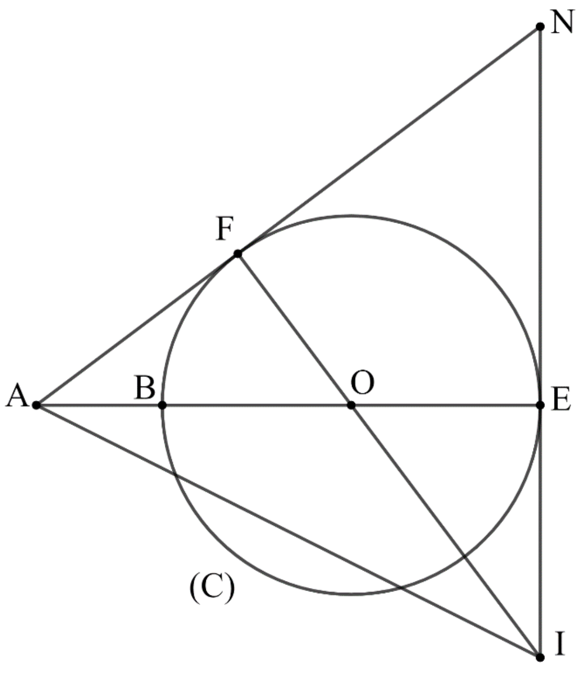
1) Plot A, B and C.
2) Prove that $y=-\frac{1}{2}x+1$ is the equation of the line (BC).
3) Calculate AC and BC. Deduce that the triangle ABC is isosceles.
4) Let (d') be the line through A and parallel to (BC).

   Prove that $y=-\frac{1}{2}x+5$ is the equation of (d').
5) Let $E(2\ ;\ 4)$ be a point on (d').
   a. Verify that [AB] and [CE] have the same midpoint.
   b. Prove that BCAE is a rhombus.

V- (5.5 points)

In the adjacent figure:

- (C) is a circle with center O and diameter BE = 6
- A is a point on (BE) so that OA = 5
- (AF) is a tangent to (C) at F
- The tangent at E to (C) intersects (AF) at N
- (NE) and (OF) intersect at I.



1) Draw the figure.
2) a. Prove that O is the orthocenter of triangle NAI.
   b. Deduce that (NO) is perpendicular to (AI).
3) a. Verify that (NO) is the perpendicular bisector of [EF].
   b. Deduce that (EF) is parallel to (AI).
4) a. Show that $\frac{OF}{OI}=\frac{OE}{OA}$.
   b. Deduce that $OI = 5$.
5) a. Prove that the four points A, F, E and I are on the same circle (C') with diameter to be determined.
   b. Verify that $AF = 4$.
   c. Calculate the radius of (C').

Question I

1	$A = \sqrt{28} - 2\sqrt{3} - \sqrt{7} + \sqrt{27} = 2\sqrt{7} - 2\sqrt{3} - \sqrt{7} + 3\sqrt{3} = \sqrt{7} + \sqrt{3}$. $B = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{7-3} = \frac{4(\sqrt{7}-\sqrt{3})}{4} = \sqrt{7} - \sqrt{3}$.	**(0.25 + 0.25 + 0.25)** **(0.5 + 0.25 + 0.25)**	**0.75** **1**
2	$A \times B = 7 - 3 = 4$.		**0.5**
3	$10m = 4$ then $m = 0.4$.	**(0.5 + 0.25)**	**0.75**

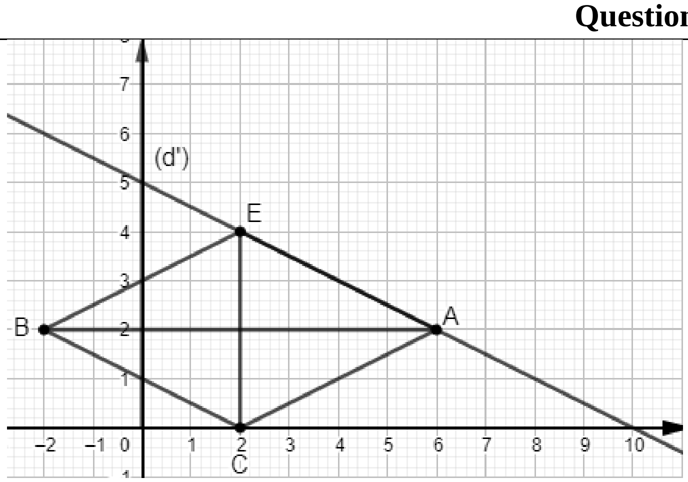
Question II

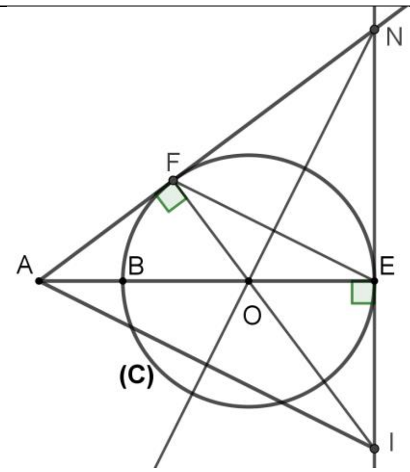
1	$A(x) = (x+9)^2 - 3(x+1)(x+9)$ $A(x) = (x+9)[(x+9) - 3(x+1)] = (x+9)(x+9-3x-3)$ $= (x+9)(-2x+6)$ $A(x) = -2(x+9)(-3+x)$	 **0.25** **0.25** **0.25**	**0.75**
2	$(x+1)(x+9) = x^2 + 9x + x + 9 = x^2 + 10x + 9$. Then : $B(x) = (x+1)(x+9)$		**0.5**
3	$A(x) = B(x)$. $-2(x+9)(x-3) = (x+1)(x+9)$ $-2(x+9)(x-3) - (x+1)(x+9) = 0$ $(x+9)[-2(x-3) - (x+1)] = 0$ $(x+9)(-2x+6-x-1) = 0$ $(x+9)(-3x+5) = 0$ then : $x = -9$ or $x = \frac{5}{3}$	 **0.25** **0.25** **0.25** **0.25 + 0.25**	**1.25**
4.a	$F(x) = \frac{-2(x-3)(x+9)}{(x+1)(x+9)}$ F(x) is defined for : $(x+1)(x+9) \neq 0$ then : $x+1 \neq 0$ and $x+9 \neq 0$ then $x \neq -1$ and $x \neq -9$	 **0.25 + 0.25**	**0.5**
4.b	$F(x) = \frac{-2(x-3)}{x+1}$		**0.25**
4.c	$\frac{-2(x-3)}{x+1} = -2$ then : $-2x + 6 = -2x - 2$; $0x = -8$ impossible	**0.25 + 0.25 + 0.25**	**0.75**

Question III

1	The number of squares $= 25\% \times 24 = 6$. The number of circles =18.	**0.75** **0.25**	**1**
2	$11 - 5 = 6$ blue circles. The number of red circles = $18 - 6 = 12$. $\frac{12}{18} = \frac{2}{3}$.	**0.25** **0.25** **0.5**	**1**

Question IV

1		**0.5 + 0.25 + 0.25**	**1**

2	$y_B = -\frac{1}{2}x_B + 1,\ y_B = 2\ ;\ -\frac{1}{2}x_B + 1 = -\frac{1}{2}(-2) + 1 = 1 + 1 = 2\,.$ then $y_B = -\frac{1}{2}x_B + 1$ so B belongs to (d). **0.5 + 0.5** similarly C belongs to (d) since $y_C = -\frac{1}{2}x_C + 1$ so $y = -\frac{1}{2}x + 1$ is the equation of (BC). Or slope (BC) = $\frac{-1}{2}$ and b = 1. **(0.75 + 0.25)**	**1**
3	$AC = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \sqrt{(2-6)^2 + (0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ **0.5** $BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(2+2)^2 + (0-2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ **0.25** AB = AC, then ABC is an isosceles triangle of vertex C. **0.25**	**1**
4	slope of (d') = slope of (d) $= -\frac{1}{2}$ and $y_A = -\frac{1}{2}x_A + b$ so $b = 5$ **0.75 + 0.25**	**1**
5.a	(2,2) is the midpoint [AB]. **0.25** (2,2) is the midpoint of [EC]. **0.5** Or slope of (BE) = slope of (AC) or AE = BC. **0.5 + 0.25**	**0.75**
5.b	AEBC is a parallelogram since the diagonals bisect each other at the same midpoint. But AC = BC, so AEBC is a rhombus or perpendicular diagonals. **0.25 + 0.5**	**0.75**
	Question V	
1		**0.5**
2.a	(IF) ⊥ (AN) and (AE) ⊥ (IN) (Tangent and radius) **0.25 + 0.25** with O is the intersection of the two heights, thus O is the orthocenter of triangle NAI. **0.25**	**0.75**
2b	(ON) is the third height so (ON) is perpendicular to (AI).	**0.25**
3a	$NE = NF$ two tangents issued from same point to same circle, and $OE = OF = 3$ **0.5 + 0.25** then (ON) is the perpendicular bisector of [EF] or (NO) is the axis of symmetry of the figure.	**0.75**
3.b	The two lines (EF) and (AI) are perpendicular to (ON) thus the two lines are parallel.	**0.5**
4.a	(EF) // (AI), then according to Thales' theorem $\frac{OF}{OI} = \frac{OE}{OA}$. **0.25 + 0.25**	**0.5**
4.b	$\frac{3}{OI} = \frac{3}{5}$ then OI = 5	**0.5**
5.a	$\widehat{AFI} = \widehat{AEI} = 90°$ then the four points A, F, E and I belong to the same circle of diameter [AI]. **0.5 + 0.25**	**0.75**
5.b	$AF^2 = OA^2 - OF^2 = 25 - 9 = 16$ then AF = 4.	**0.5**
5.c	$AI^2 = AF^2 + FI^2 = 16 + 64 = 80$ then $AI = 4\sqrt{5}$; so the radius $= 2\sqrt{5}$. **0.25 + 0.25**	**0.5**