عدد المسائل: خمس　　مسابقة في مادة الرياضيات　　الاسم:
المدة: ساعتان　　الرقم:

إرشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I – (2.5 points)

Consider the three numbers A, B and C so that:

$A=\sqrt{18}-\sqrt{8}+\sqrt{50}$, $B=\dfrac{1}{\sqrt{2}+1}$ and $C=\left(\sqrt{2}+1\right)^2+1$.

(Show all the steps of calculation)

1) Write A in the form $m\sqrt{2}$ where m is an integer.

2) Show that $B=\sqrt{2}-1$.

3) Write C in the form $n\sqrt{2}+p$ where n and p are integers.

4) Show that $B\times A\times C$ is an integer.

II – (4 points)

1) Given $P(x)=(2x+1)^2-2x^2-9x-4$

a. Verify that $(2x+1)(x+4)=2x^2+9x+4$.

b. Show that $P(x)=(2x+1)(x-3)$.

c. Solve the equation $P(x)=0$.

2) Let $H(x)=\dfrac{P(x)}{4x^2-1}$.

a. Factorize $4x^2-1$.

b. For what values of x, is $H(x)$ defined?

c. Simplify $H(x)$.

3) Let ABC be a right triangle at A so that $AB=x-3$ and $BC=2x-1$ *where* $x>3$.

a. Verify that $\sin \widehat{BCA}=H(x)$.

b. Is there a value of x so that $\widehat{BCA}=30°$? Justify.

III – (3 points)

1) Solve the following system: $\begin{cases} x+y=16 \\ 2x+3y=38 \end{cases}$

2) The following table represents the distribution of electronic games in a shop according to their prices:

Price of an electronic game (in LL)	3 000	4 000	5 000	6 000
Number of electronic games	9	m	15	n

a. The total price of all electronic games in this shop is 178 000 LL.
Show that this information is modeled by the following equation: $2m+3n=38$.

b. Knowing that the total number of electronic games in this shop is 40. Calculate m and n.

3) Given $m=10$ and $n=6$.
Calculate the average price (mean) of these 40 electronic games.

IV- (5.5 points)

In an orthonormal system of axes $x'Ox$ and $y'Oy$, given the points $A(0;-2)$, $B(-4;0)$ and $C(0;3)$.

1) a. Plot the points A, B and C.

b. Show that the equation of line (AB) is $y=\frac{-1}{2}x-2$.

2) Show that triangle ABC is isosceles of vertex C.

3) Let H be the point with coordinates $(-2;-1)$.

a. Verify that H is the midpoint of [AB].

b. Determine the equation of the bisector of angle BCA.

4) a. Prove that the points B, H, O and C are on the same circle of center $I\left(-2;\frac{3}{2}\right)$ and calculate its radius.

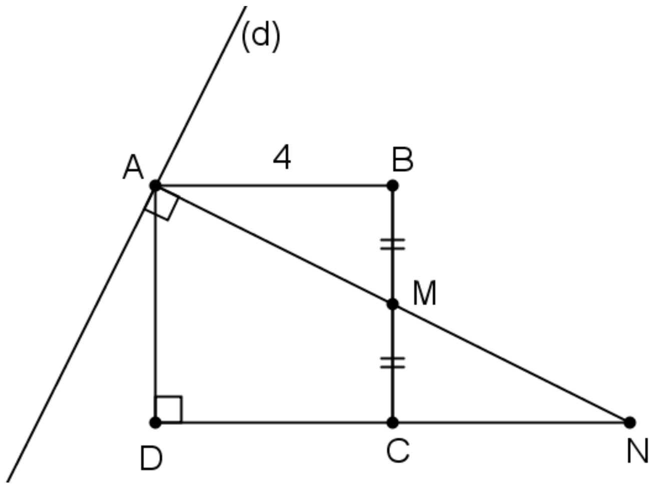
b. Show that (IH) is parallel to the y-axis.

5) Let K be the point of coordinates $(-2;0)$. Calculate the area of trapezoid HACI.

V- (5 points)

In the adjacent figure:

- ABCD is a square of side 4
- M is the midpoint of [BC]
- (AM) intersects (DC) at N
- (d) is the perpendicular through A to (AM).



1) Reproduce the figure.

2) Calculate AM.

3) Calculate the ratio $\frac{NC}{ND}$, deduce that C is the midpoint of [DN].

4) Lines (d) and (CD) intersect at Q.

a. Show that $AQD = NAD$.

b. Show that the two triangles DAQ and DNA are similar.

Deduce that $DQ \times DN = 16$.

c. Calculate DQ.

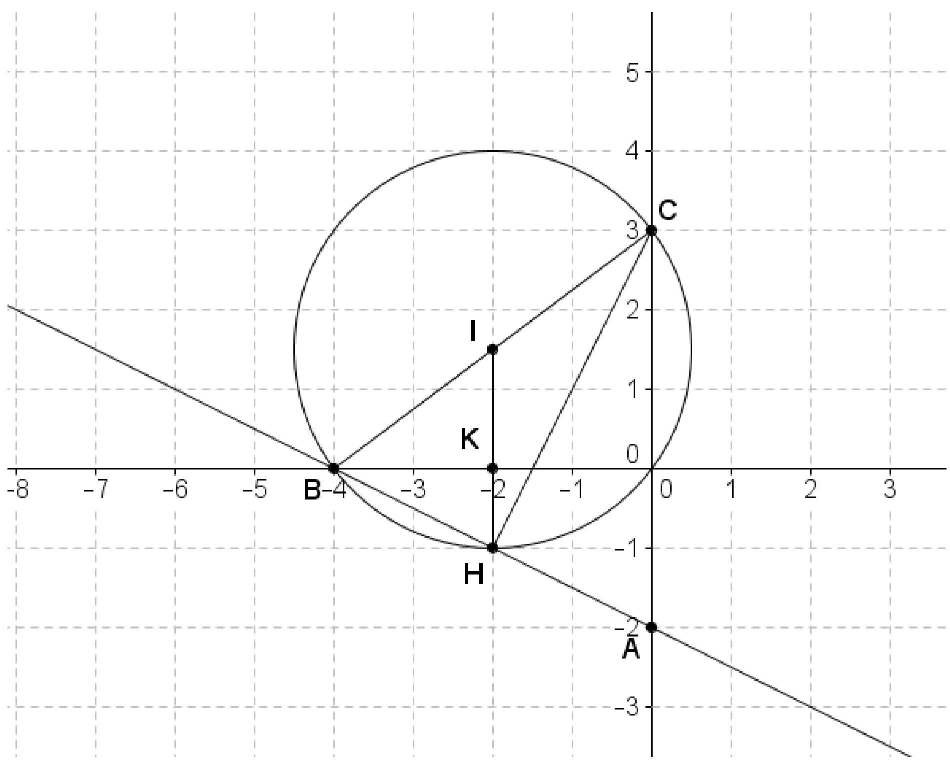
5) Show that the triangle AQM is a right isosceles triangle at A.

6) Let (C) be the circle with diameter [AQ] and L is the translate of Q under the translation with vector $\overrightarrow{AM}$.

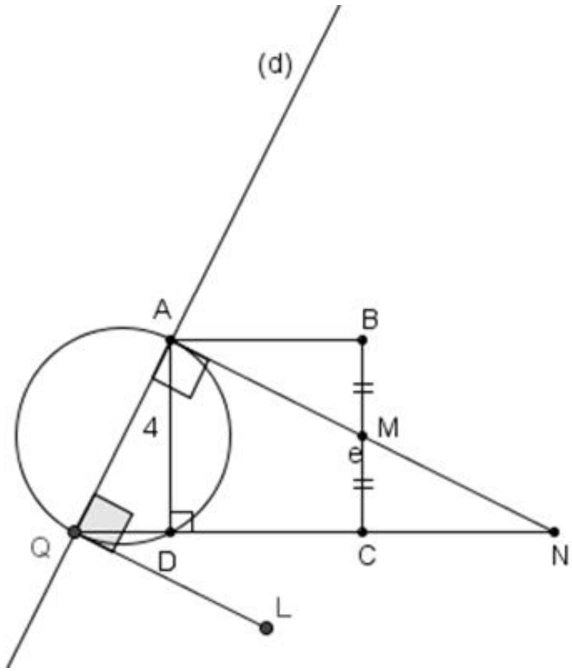
Show that (LQ) is tangent to circle (C).

Part	Answer Key	Grade
Question I		
1	$A = \sqrt{18} - \sqrt{8} + \sqrt{50}$ $A = 3\sqrt{2} - 2\sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$. **0.25 + 0.25**	**0.5**
2	$B = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \sqrt{2} - 1$. **0.25 + 0.5**	**0.75**
3	$C = \left(\sqrt{2}+1\right)^2 + 1 = 2 + 1 + 2\sqrt{2} + 1 = 4 + 2\sqrt{2}$ **0.5 + 0.25**	**0.75**
4	$B \times A \times C = (\sqrt{2}-1)(6\sqrt{2})(4+2\sqrt{2}) = 12(2-\sqrt{2})(2+\sqrt{2}) = 24$ **0.25 + 0.25**	**0.5**
Question II		
1a	$(2x+1)(x+4) = 2x^2 + 8x + x + 4 = 2x^2 + 9x + 4$. **0.25 + 0.25**	**0.5**
1b	$P(x) = (2x+1)^2 - 2x^2 - 9x - 4$ $P(x) = (2x+1)^2 - (2x^2 + 9x + 4)$ **0.25** $P(x) = (2x+1)^2 - (2x+1)(x+4)$ $= (2x+1)[(2x+1) - (x+4)]$ **0.25** $= (2x+1)(x-3)$ **or expand both expressions. 0.25**	**0.75**
1c	$P(x) = 0$ $(2x+1)(x-3) = 0$ gives : $x = -\frac{1}{2}$ or $x = 3$ **0.25 + 0.25**	**0.5**
2a	$4x^2 - 1 = (2x-1)(2x+1)$	**0.5**
2b	$H(x) = \frac{P(x)}{4x^2-1} = \frac{(2x+1)(x-3)}{(2x-1)(2x+1)}$ $H(x)$ *is defined if* $(2x-1)(2x+1) \neq 0$ $x \neq \frac{1}{2}$ and $x \neq -\frac{1}{2}$ **0.25 + 0.25**	**0.5**
2c	$H(x) = \frac{(2x+1)(x-3)}{(2x-1)(2x+1)} = \frac{x-3}{2x+1}$	**0.25**
3a	$\sin \text{BCA} = \frac{AB}{BC} = \frac{x-3}{2x-1}$ **0.25 + 0.25**	**0.5**
3b	$\sin 30^0 = \frac{x-3}{2x-1}$ $\frac{1}{2} = \frac{x-3}{2x-1}$ **0.25** $0 = 5$ impossible **0.25**	**0.5**

Question III		
1	$\begin{cases} x+y=16 \\ 2x+3y=38 \end{cases}$ $x=10$; $y=6$ **0.5 + 0.5**	**1**
2a	$3000\times 9+4000\times m+5000\times 15+6000\times n=178\,000$ **0.5** $2m+3n=38$ **0.25**	**0.75**
2b	$9+m+15+n=40$ $m+n=16$ **0.25** Hence the system $\begin{cases} m+n=16 \\ 2m+3n=38 \end{cases}$ Using part 1) m = 10 and n = 6. **0.5**	**0.75**
3	The average price of an electronic game is $=\frac{178\,000}{40}=4\,450$ LL	**0.5**

Question IV		
1a		**0.5**

1b	$a_{(AB)} = \frac{y_B - y_A}{x_B - x_A} = \frac{0+2}{-4-0} = -\frac{1}{2}$ **0.5** $y = -\frac{1}{2}x + b$ $y_A = -\frac{1}{2}x_A + b$ $b = -2$ **0.25** Then the equation of line (AB) is $y = -\frac{1}{2}x - 2$	**0.75**
2	$CB = \sqrt{(-4-0)^2 + (0-3)^2} = 5$ **0.5** $CA = \sqrt{(0-0)^2 + (-2-3)^2} = 5$ **0.25** CA = CB Triangle ABC is isosceles of vertex C.	**0.75**
3a	$x_H = \frac{x_A + x_B}{2}$ $-2 = \frac{0-4}{2}$ $-2 = -2$ **0.25** $y_H = \frac{y_A + y_B}{2}$ **0.25** $-1 = \frac{-2+0}{2}$ $-1 = -1$ then H is the midpoint of [AB].	**0.5**
3b	In triangle ABC isosceles of vertex C, the bisector is also the median which is (CH). $a_{(CH)} = \frac{y_H - y_C}{x_H - x_C} = 2$ **0.5** $y = 2x + b$ $y_H = 2x_H + b$ **0.25** $b = 3$ then the equation of the bisector (CH) is $y = 2x + 3$	**0.75**
4a	BOC and BHC are two right triangles, then they can be inscribed in a circle of diameter [BC] their common hypotenuse. So the four points B, H, O and C are on the same circle of center I midpoint of [BC]. **0.25** $x_I = \frac{x_B + x_c}{2}$ $-2 = \frac{-4+0}{2}$ $-2 = -2$ **0.25** $y_I = \frac{y_B + y_C}{2}$ $\frac{3}{2} = \frac{0+3}{2}$ $\frac{3}{2} = \frac{3}{2}$ **0.25** Radius = $\frac{CB}{2} = \frac{5}{2}$. **0.25**	**1**
4b	We have : $x_I = x_H = -2$ so (IH) : $x = -2$ // y'oy **0.25 + 0.25**	**0.5**
5	**Area** = $\frac{OK \times (IH+AC)}{2} = \frac{2\times(2.5+5)}{2} = 7.5$ **0.25 + 0.25 + 0.25**	**0.75**

	Question V	
1		**0.5**
2	AMB is a right triangle. $AM^2 = AB^2 + BM^2 = 16 + 4 = 20$ (Pythagorean) **0.25** $AM = \sqrt{20} = 2\sqrt{5}$ **0.25**	**0.5**
3	We have (MC) // (AD) Using Thales' theorem: $\frac{NC}{ND} = \frac{CM}{AD} = \frac{1}{2}$ **0.25 + 0.25** Then ND = 2 NC so C is the midpoint [DN]. **0.25**	**0.75**
4a	$\widehat{AQD} = \widehat{NAD}$ having the same complement $\widehat{QAD}$	**0.25**
4b	In triangles DAQ and DNA we have : $\widehat{AQD} = \widehat{NAD}$ (proved) **0.25** $\widehat{ADQ} = \widehat{ADN} = 90^0$ **0.5** $\frac{DA}{DN} = \frac{DQ}{DA}$ (ratio of similitude) **0.5** $DN \times DQ = DA^2 = 4^2 = 16$ **0.25**	**1.5**
4c	DN = 8 since C is the midpoint of [DN]. then $8 \times DQ = 16$ therefore DQ = 2	**0.25**
5	In triangle ADQ : $AQ^2 = AD^2 + DQ^2 = 16 + 4 = 20$ (Pythagorean) $AQ = \sqrt{20} = 2\sqrt{5}$ = AM **0.5** Then AMQ is isosceles. and $\widehat{QAM} = 90^0$ **0.25** Then : AMQ is a right isosceles triangle at A.	**0.75**
6	$\overrightarrow{QL} = \overrightarrow{AM}$ then MLQA is a parallelogram, having a right angle then it is a rectangle so (QL) ⊥ (AQ) and (QL) will be a tangent to (C).	**0.5**