وزارة التربية والتعليم العالي المديريّــة العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: الإجتماع والإقتصاد	دورة الـعام ٢٠١٨ العاديّة الاثنين ١١ حـزيران ٢٠١٨ مكفوفين

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

A restaurant distributes brochures each month for advertisement.
The table below shows the number of distributed brochures (y_i) in thousands and the monthly cost of distribution (x_i) in hundred thousands LL.

Month	January 2018	February 2018	March 2018	April 2018	May 2018	June 2018
Cost of distribution (x_i) in hundred thousands LL	1	3.5	2	5	1.5	2.4
Number of distributed brochures (y_i) in thousands	1.2	6.4	2.6	7.2	2.1	3.2

1) Find the coordinates of the center of gravity $G(\bar{x}, \bar{y})$.
2) Write an equation of the regression line $(D_{y/x})$.
3) Find the correlation coefficient r and interpret its value.
4) The above model remains valid in the year 2018.
   The restaurant manager receives an advertisement offer for the month of July 2018.
   The offer indicates: “4 000 distributed brochures for only 250 000 LL”.
   Is it more advantageous for the manager to take this offer or to remain using the given model? Justify.

II- (4 points)

In 2017, the students of the third secondary classes of a certain school are distributed as follows:

- 50% of the students are in the ES section of which 60% succeeded in the official exam.
- 10% of the students are in the GS section of which 80% succeeded in the official exam.
- 40% of the students are in the LS section.
- 60% of the students succeeded in the official exam.

Part A

One student is randomly selected from the third secondary students of this school.
Consider the following events:
E: “The selected student is in the ES section”, G: “The selected student is in the GS section”,
L: “The selected student is in the LS section”, S: “The selected student succeeded in the official exam”.

1) a- Calculate the probabilities $P(E \cap S)$ and $P(G \cap S)$.
   b- Prove that $P(L \cap S) = 0.22$.
2) The selected student succeeded in the official exam. Calculate the probability that this student is in the LS section.

Part B

There are 50 students in the third secondary classes in this school in 2017. A computer software selects randomly and simultaneously the names of three students from the 50 students.

1) Verify that 30 students of this school succeeded in the official exam.
2) Let X be the random variable equal to the number of students who succeeded in the official exam among the three selected names of the students.
   a- Calculate $P(X = 1)$.
   b- Calculate the probability of selecting at least one name of a student who succeeded in the official exam.

III- (4 points)

At the beginning of the year 2015, Nabil deposits a capital of 60 million LL in a bank, at an annual interest rate of 6% , compounded annually.

At the beginning of every year, and after compounding the interest, Nabil deposits an additional amount of 3 000 000 LL in the same account.

For all natural numbers n, denote by S_n the amount, in millions LL, that Nabil has in his account at the end of the year (2015 + n).

Thus, $S_0 = 60$ and $S_{n+1} = 1.06S_n + 3$ for all natural numbers n.

1) Calculate the amount of money in Nabil's account at the end of the year 2016.
2) Let (V_n) be the sequence defined as $V_n = S_n + 50$ for all natural numbers n.
   a- Show that (V_n) is a geometric sequence whose common ratio and first term V_0 are to be determined.
   b- Show that $S_n = 110 \times (1.06)^n - 50$ for all natural numbers n.
   c- Show that the sequence (S_n) is strictly increasing.
3) Calculate the amount of money in Nabil's account at the end of the year 2020.
4) Nabil wants to buy a piece of land that costs 90 million LL.
   In which year would Nabil be able, for the first time, to buy this piece of land? Justify.

IV-(8 points)

Consider the function f defined over the interval I = [1 , + ∞[as $f(x) = (10x - 10)\, e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

1) Determine $\lim_{x \to +\infty} f(x)$ and deduce an asymptote to (C).
2) Show that $f'(x) = 10(-x+2)e^{-x}$. Study the sense of variations of f and specify the maximum of f.
3) Let F be the function defined, over I, as $F(x) = -10\,x\,e^{-x}$. Prove that F is an antiderivative of f.

Part B

A company produces a certain type of objects.

The demand function f and the supply function g, defined over J = [2 , 10], are respectively modeled as $f(x) = (10x - 10)\, e^{-x}$ and $g(x) = e^{x-4}$, where f(x) and g(x) are expressed in thousands of objects and the unit price x is expressed in millions of LL. *(The unit price is the price of 1000 objects)*

1) Calculate the number of demanded objects for a unit price of 3 000 000 LL.
2) Find the unit price for a supply of 1 000 objects.
3) The equation f(x) = g(x) has, over J, a unique solution α.
   Suppose that α = 3.635.
   a- Give an economical interpretation of α and calculate the corresponding number of objects.
   b- Calculate, in LL, the revenue corresponding to the value of α given above.
4) Denote by E(x) the elasticity of the demand with respect to the unit price x.
   a- Show that $E(x) = \dfrac{x^2 - 2x}{x - 1}$.
   b- For an increase of 1% on the unit price x_0 in millions LL, the demand will decrease by 1.5%. Calculate x_0.