وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع العلوم العامة

دورة سنة ٢٠٠٤ الاكمالية الاستثنائية

مسابقة في الرياضيات
المدة: ٤ ساعات

الاسم:
الرقم:

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (2 points) .

In the space referred to a direct orthonormal system $(O\,;\vec{i}\,,\,\vec{j}\,,\vec{k}\,)$,

consider the line (d) defined by $:\begin{cases} x = t+1 \\ y = -t+2 \\ z = 2t \end{cases}$ (t is a real parameter)

and the plane (P) of equation $x - y - 2z - 5 = 0$.

1) Determine the coordinates of E, the point of intersection of (d) and (P).

2) a- Write an equation of the plane (Q) perpendicular to (d) at E.
   b- Find a system of parametric equations of the line (D) that lies in plane (P) and is perpendicular to (d) at E.

3) I(2 ; 1 ; 2) is a point on line (d) . Determine the coordinates of J, the symmetric of I with respect to (D).

II-(3.5 points)

In the plane referred to an orthonormal system $(O;\ \vec{i}\ ,\ \vec{j}\ )$, consider the conic (C_m) of equation : $2mx^2 + (m+1)y^2 - 8(m-1)x - 2m - 1 = 0$ where m is a real parameter different from -1.

1) For what value of m would the conic (C_m) be a parabola ?
Determine the vertex, the focus and the directrix of this parabola.

2) In this question, let $m = 2$.

a- Determine the nature and the center of (C_2), and find its vertices that lie on the focal axis.

b- The conic (C_2) cuts the axis of ordinates at the points G and L .
Write the equations of the tangents to (C_2) at these two points.

c- Calculate the area of the region bounded between (C_2) and its principal circle .

3) Let f be the function expressed by $f(x)=\sqrt{\frac{3}{2}-x^2}$,and designate by (T) its representative curve in the system $(O;\ \vec{i}\ ,\ \vec{j}\ )$.

a- Prove that (T) is a part of one of the curves (C_m) .Determine, in this case, the nature and elements of (C_m) .

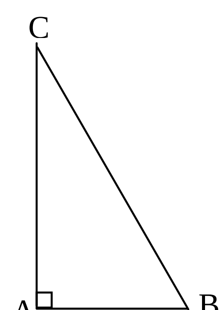
b- Let (D) designate the region bounded between (T) and the axis of abscissas.
Calculate the volume of the solid of revolution generated by the rotation of the region (D) about the axis of abscissas.

III- (2 points)

In an oriented plane, consider a direct triangle ABC right angled at A, such that

AB = 2cm and $(\vec{BC}\ ;\ \vec{BA}\ ) = \frac{\pi}{3}\ (2\pi)$.

Let **S** be the direct similitude that transforms A onto B and B onto C.



1) Determine the ratio and the angle of **S**.

2) a- Construct the point C', the image of C under **S**.
(Give the steps of this construction).
b- Calculate the area of triangle BCC'.

3) Let O be the midpoint [AB] , and consider the direct orthonormal system
$(O;\vec{u},\vec{v})$ such that $\vec{u} = \vec{OB}$.
a- Find the complex form of the similitude **S**.
b- Determine the affix of point W , the center of **S** .
c- Let $\mathbf{S}^{-1}$ be the inverse transformation of **S** . Give the complex form of $\mathbf{S}^{-1}$.

IV- (2.5 points)

The complex plane is referred to a direct orthonormal system $(O;\vec{u},\vec{v})$.
A, B and C designate three points, in this plane, whose affixes are a, b and c
respectively.

1) Show that if the triangle ABC is right angled at B, then the complex number $\frac{c-b}{a-b}$
is pure imaginary.

2) In this question, suppose that $a = z$, $b = z^2$ and $c = z^4$.
a- Solve the equation $z^4 - z = 0$.
b- Find the values of z for which the points A, B and C are distinct in pairs.
c- Prove that when the triangle ABC is right angled at B, then the point A of affix
$z = x + iy$ moves on a conic whose equation and nature are to be determined .

V- (3,5 points)

An urn contains nine balls :

three white, numbered from 1 to 3
three black, numbered from 1 to 3
three red , numbered from 1 to 3.

Two balls are drawn, simultaneously and at random, from this urn.
Consider the following events :

A : " the **two** drawn balls carry odd numbers " .
B : " The **two** drawn balls have the same colour " .
C : " The **two** drawn balls have different colours " .
D : " The **two** drawn balls have different colours and carry two odd numbers ".

1) Calculate the following probabilities : P(A) , P(B) , $P(A \cap B)$ and P(A/B).
Are the events A and B independent ?

2) a- Calculate P(C) and prove that $P(D) = \frac{1}{3}$.

b- The two drawn balls are of different colours, what is the probability that they carry two odd numbers ?

3) Let X be the random variable, ($X \geq 0$), that is equal to the absolute value of the difference between the numbers carried by the two drawn balls.
Determine the probability distribution of X and calculate the mathematical expectancy E(X).

VI- (7 points)

Let f_n be the function defined on IR by $f_n(x) = \frac{2e^{nx}}{1 + e^x} - 1$, where n is a natural integer, and let (C_n) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$. *Unit 2 cm.*

A- Suppose, in this part, that $n = 1$:

1) Calculate $\lim_{x \to +\infty} f_1(x)$ and $\lim_{x \to -\infty} f_1(x)$.

2) Calculate $f'_1(x)$ and set up the table of variations of f_1 .

3) a- Prove that O is a point of inflection of (C_1).

   b- Write an equation of the line (d) that is tangent to O to (C_1) .

4) Draw (d) and (C_1) .

B- Let (C_0) be the representative curve of the function f_0 , corresponding to $n = 0$, in the same system $(O; \vec{i}, \vec{j})$.

1) Prove that (C_0) is the symmetric of (C_1) with respect to the axis of ordinates.

2) Prove that (C_0) is the symmetric of (C_1) with respect to the axis of abscissas.

3) Calculate , in cm^2, the area of the region bounded between (C_1), (C_0) and the lines of equation $x = 0$ and $x = 1$.

C- Consider the sequence (U_n) defined by $U_n = \int_0^1 f_n(x)\,dx$.

1) Prove that $U_{n+1} + U_n = 2\dfrac{e^n - n - 1}{n}$.

2) Calculate $\lim_{n \to +\infty}(U_{n+1} + U_n)$ and deduce that the sequence (U_n) cannot be convergent.

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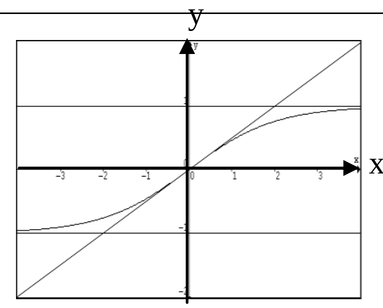
Questions		Answers	G
I	1	t +1 + t – 2 – 4t – 5 = 0 ; t = – 3 ; E(–2 ; 5 ; –6)	½
	2-a-	M(x ; y ; z) is a point of (Q) iff $\overrightarrow{EM} \cdot \overrightarrow{V}_d = 0$; (Q) : x – y + 2z + 19 = 0	1
	2-b-	The line (D), of (P), passes through E and it is perpendicular to (d), it is contained in (Q), it is the line of intersection of (P) and (Q). (D) : x = t – 7 ; y = t , z = – 6	1 ½
	3	(d) is perpendicular to (D) at E , then J is the symmetric of I with respect to E ;so E is the mid point of [IJ], then J(– 6 ; 9 ; –14).	1
II	1	(C_m) is a parabola iff 2m (m + 1) = 0 , then m = 0 (since m ≠ –1) (C_o) : $y^2 + 8x - 1 = 0$; $y^2 = -8(x - \frac{1}{8})$; p = 4 Vertex V($\frac{1}{8}$; 0) ; focus F($-\frac{15}{8}$; 0) ; directrice x = $\frac{17}{8}$	1 ½
	2-a-	(C_2) : $4x^2 + 3y^2 - 8x - 5 = 0$; $\frac{(x-1)^2}{\frac{9}{4}} + \frac{y^2}{3} = 1$; a = $\sqrt{3}$ and b = $\frac{3}{2}$ Ellipse ;Center O'(1 ; 0) ; principal vertices (1, $\sqrt{3}$) and (1 ; $-\sqrt{3}$).	1
	2-b-	If x = 0 then y = $\sqrt{\frac{5}{3}}$ or y = $-\sqrt{\frac{5}{3}}$; G(0 ; $\sqrt{\frac{5}{3}}$) and L(0 ; $-\sqrt{\frac{5}{3}}$) 8x + 6yy' – 8 = 0 ; then y' = $\frac{4-4x}{3y}$. $y'_G = \frac{4}{\sqrt{15}}$ and $y'_L = -\frac{4}{\sqrt{15}}$ (T_G) : y = $\frac{4}{\sqrt{15}}x + \sqrt{\frac{5}{3}}$ and (T_L) : y = $-\frac{4}{\sqrt{15}}x - \sqrt{\frac{5}{3}}$	1
	2-c-	A = area(principal circle) – area(C_2) = $\pi a^2 - \pi ab = 3\pi(1 - \frac{\sqrt{3}}{2})$ u^2	1
	3-a-	(T) : y = $\sqrt{\frac{3}{2} - x^2}$; (T) is a part of the curve of equation $y^2 = \frac{3}{2} - x^2$ or $x^2 + y^2 = \frac{3}{2}$ that is the equation of (C_1). (T) is a part of a circle (C_1) of center O and of radius r = $\sqrt{\frac{3}{2}}$.	1 ½
	3-b-	V = $\pi \int_{-\sqrt{\frac{3}{2}}}^{\sqrt{\frac{3}{2}}} f^2(x)dx = 2\pi \int_0^{\sqrt{\frac{3}{2}}} (\frac{3}{2} - x^2)dx = 2\pi \left[\frac{3}{2}x - \frac{x^3}{3}\right]_0^{\sqrt{\frac{3}{2}}} = \pi\sqrt{6}$ u^3 ►or : (T) is a semi circle, then the solid generated by the rotation of (D) about the axis of abscissas is a sphere of center O and of radius	1

		$r=\sqrt{\frac{3}{2}}$; $V=\frac{4}{3}\pi r^3=\pi\sqrt{6}\ u^3$	
III	1	S(A) = B ; S(B) = C . ABC is a semi-equilateral triangle then BC = 4 cm. $k=\frac{BC}{AB}=2$; $\alpha=(\overrightarrow{AB},\overrightarrow{BC})=\frac{2\pi}{3}\ (2\pi)$	1
	2-a-	S(B) = C and S(C) = C’, CC’ = k BC = 2BC = 8cm $(\overrightarrow{BC},\overrightarrow{CC'})=\frac{2\pi}{3}$; then $(\overrightarrow{CB},\overrightarrow{CC'})=\frac{2\pi}{3}-\pi=-\frac{\pi}{3}$. C’ is the point of intersection of the circle (C ; 8) and the semi-line [Ct) such that $(\overrightarrow{CB},\overrightarrow{Ct})=-\frac{\pi}{3}$. ►Or : The triangle BCC’ is directly similar to ABC, it is semi equilateral with $(\overrightarrow{BC},\overrightarrow{BC'})=\frac{\pi}{2}$.	1
	2-b-	S(B) = C , S(A) = B and S(C) = C’ , area(BCC’) = $k^2\times$ area(ABC) = $4\times\frac{1}{2}AB\times AC=8\sqrt{3}\ cm^2$.	1
	3-a-	$z_A=-1$, $z_B=1$; $z'=2e^{i\frac{2\pi}{3}}z+b=(-1+i\sqrt{3})z+b$ S(A) = B gives $b=i\sqrt{3}$; then $z'=(-1+i\sqrt{3})z+i\sqrt{3}$.	1
	3-b-	$z_w=(-1+i\sqrt{3})z_w+i\sqrt{3}$; $z_W=-\frac{3}{7}+\frac{2i\sqrt{3}}{7}$.	½
	3-c-	$z=\frac{z'-i\sqrt{3}}{-1+i\sqrt{3}}=(\frac{-1-i\sqrt{3}}{4})z'-\frac{3}{4}+i\frac{\sqrt{3}}{4}$ The complex form of S^{-1} is : $z'=(\frac{-1-i\sqrt{3}}{4})z-\frac{3}{4}+i\frac{\sqrt{3}}{4}$. ►Or : S^{-1} is the similitude (W ; $\frac{1}{2}$;$-\frac{2\pi}{3}$) ; $z'-z_w=\frac{1}{2}e^{-i\frac{2\pi}{3}}(z-z_W)$	½
IV	1	If ABC is right at B , then $(\overrightarrow{BA},\overrightarrow{BC})=\frac{\pi}{2}\ (\pi)$, and $\arg(\frac{z_{\overrightarrow{BC}}}{z_{\overrightarrow{BA}}})=\frac{\pi}{2}\ (\pi)$ so $\frac{z_C-z_B}{z_A-z_B}$ is pure imaginairy, i.e $\frac{c-b}{a-b}$ is pure imaginairy.	1
	2-a	$z^4-z=0$ is equivalent to $z(z^3-1)=0$ $z=0$, $z=1$, $z=-\frac{1}{2}+i\frac{\sqrt{3}}{2}=j$ or $z=-\frac{1}{2}-i\frac{\sqrt{3}}{2}=j^2$.	½
	2-b	A , B et C are distinct in pairs iff $z\neq z^2$, $z\neq z^4$ et $z^2\neq z^4$	1

		$z \in \mathbb{C} - \left\{0\,,\,1\,,\,-1\,,\,-\frac{1}{2}+i\frac{\sqrt{3}}{2}\,,-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right\}$	
	2-c-	The triangle ABC is right at B hence $\frac{z^4-z^2}{z-z^2}$ is pure imaginairy ; $\frac{z^4-z^2}{z-z^2} = -z(z+1) = -x^2+y^2-x-i(2xy+y)$ $\frac{z^4-z^2}{z-z^2}$ is pure imaginairy iff $-x^2+y^2-x=0$ and $2xy+y \neq 0$ A moves on a hyperbola of equation $x^2-y^2+x=0$.	1 ½
	1	Number of possible cases $C_9^2 = 36$ $P(A) = \frac{C_6^2}{C_9^2} = \frac{15}{36} = \frac{5}{12}$ $\begin{pmatrix} b_1 & b_2 & b_3 \\ w_1 & w_2 & w_3 \\ r_1 & r_2 & r_3 \end{pmatrix}$ $P(B) = P(2b)+P(2w)+p(2r) = \frac{3C_3^2}{C_9^2} = \frac{9}{36} = \frac{1}{4}$ $A \cap B = \{ b_1b_3 ; w_1w_3 ; r_1r_3\}$; $P(A \cap B) = \frac{3}{36} = \frac{1}{12}$. $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{1/4} = \frac{1}{3}$ $P(A/B) \neq P(A)$, then A and B are not independent.	2
IV	2-a-	$C = \overline{B}$, so $P(C) = 1 - P(B) = \frac{3}{4}$. $D = A \cap \overline{B}$; $p(D) = P(A) - P(A \cap B) = \frac{5}{12} - \frac{1}{12} = \frac{1}{3}$ ►Or: We compute 12 favorable outcomes among 36 ; $P(D) = \frac{12}{36} = \frac{1}{3}$	1
	2-b-	$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{P(D)}{P(C)} = \frac{1/3}{3/4} = \frac{4}{9}$.	1
	3	The possible values of X are 0 ; 1 ; 2. $P(X=0)$ = P(the two balls having the same number) $= \frac{3C_3^2}{C_9^2} = \frac{9}{36} = \frac{1}{4}$ $P(X=2) = \frac{C_3^1 \times C_3^1}{C_9^2} = \frac{9}{36} = \frac{1}{4}$; $P(X=1) = 1 - (\frac{1}{4} + \frac{1}{4}) = 1 - \frac{1}{2} = \frac{1}{2}$ $X = x_i$: 0 ; 1 ; 2 p_i : $\frac{1}{4}$; $\frac{1}{2}$; $\frac{1}{4}$ $E(X) = 1$.	2

$X = x_i$	0	1	2
p_i	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

VI	A-1-	$f_1(x)=\dfrac{2e^x}{1+e^x}-1$ $\lim\limits_{x\to+\infty} f_1(x)=\lim\limits_{x\to+\infty}\dfrac{2}{e^{-x}+1}-1=2-1=1$; $\lim\limits_{x\to-\infty} f_1(x)=-1$	1
	A-2-	$f_1'(x)=\dfrac{2e^x}{(1+e^x)^2}$ x: $-\infty$ … $+\infty$ $f_1'(x)$: + $f_1(x)$: -1 ↗ 1	1 ½
	A-3-a	$f_1''(x)=\dfrac{2e^x(1-e^x)}{(1+e^x)^4}$; $f_1''(0)=0$; $f_1''(x)>0$ for $x<0$ and $f_1''(x)<0$ for $x>0$, hence O(0 ;0) is a point of inflection of(C_1).	1
	A-3-b	(d) : $y=f_1'(0).x=\dfrac{1}{2}x$.	1
	A-4	y =1 :H.A y = –1 :H.A 	2
	B-1-	$f_0(x)=\dfrac{2}{1+e^x}-1$; $f_1(-x)=\dfrac{2e^{-x}}{1+e^{-x}}-1=\dfrac{2}{e^x+1}-1=f_0(x)$, So, the curve (C_0) is symmetric of (C_1) with respect to the axis of ordinates.	1
	B-2-	$f_1(x)+f_0(x)=\dfrac{2e^x}{1+e^x}-1+\dfrac{2}{1+e^x}-1=\dfrac{2(e^x+1)}{1+e^x}-2=0$, So, (C_0) is symmetric of (C_1) with respect to the axis of abscissas.	1
	B-3-	The required area is twice the area of the region bounded by (C_1) , the lines: x = 0 , x = 1 and the axis of abscissas. $\int_0^1 f_1(x)dx=\int_0^1(\dfrac{2e^x}{1+e^x}-1).dx=\left[2\ln(1+e^x)-x\right]_0^1=2\ln\dfrac{1+e}{2}-1$ A = 2 $\int_0^1 f_1(x).dx\ u^2=(4\ln\dfrac{1+e}{2}-2)u^2=(16\ln\dfrac{1+e}{2}-8)\ cm^2$	1 ½
	C-1-	$u_{n+1}+u_n=\int_0^1(\dfrac{2e^{(n+1)x}}{e^x+1}+\dfrac{2e^{nx}}{e^x+1}-2)dx=\int_0^1[\dfrac{2e^{nx}(e^x+1)}{e^x+1}-2].dx$ $=\int_0^1[2e^{nx}-2].dx=\left[\dfrac{2}{n}e^{nx}-2x\right]_0^1=2\dfrac{e^n-n-1}{n}$.	2

	C-2-	$\lim_{n\to+\infty}(u_{n+1}+u_n)=\lim_{n\to+\infty}(2\frac{e^n}{n}-2-\frac{2}{n})=+\infty$. If (u_n) converges to a finite number ℓ then $(u_{n+1}+u_n)$ converges to 2ℓ ; which is impossible.	2