وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٦ العادية

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I– **(4 points)**

In the space referred to a direct orthonormal system $(O\,;\ \vec{i}\ ,\ \vec{j}\ ,\ \vec{k})$, consider the plane (P) of equation $x + y + z - 4 = 0$ and the points A (3 ;1 ; 0) , B(1; 2 ;1) , C(1 ; 1 ;2) and E(2 ; 0 ; –1).

1) Prove that the triangle ABC is right angled at B.
2) a- Verify that the points A, B and C belong to the plane (P).
   b- Write a system of parametric equations of the line (d) that is perpendicular to plane (P) at point A, and verify that the point E belongs to (d).
3) Designate by (Q) the plane that passes through A and is perpendicular to (BE). Write an equation of (Q).
4) The planes (P) and (Q) intersect along a line (D).
   a- Prove that the lines (D) and (BC) are parallel.
   b- M is a variable point on (BC), prove that the distance from M to plane (Q) remains constant.

II– **(4.5 points)**

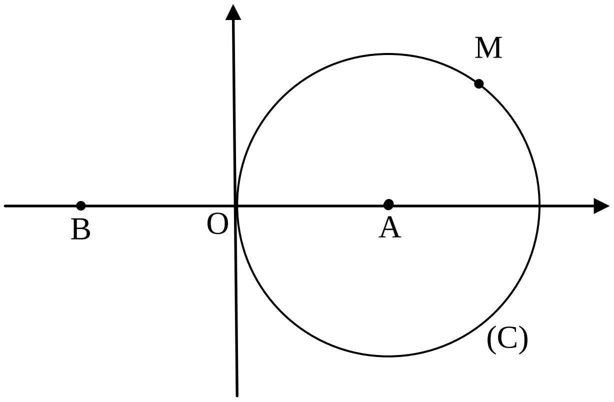
A bag **S** contains **eight** bills: **four** bills of 10 000LL, **three** of 20 000LL and **one** of 50 000LL.
Another bag **T** contains also **eight** bills : **three** bills of 10 000LL and **five** of 20 000LL.

1) **Two bills are drawn, simultaneously and randomly, from the bag S**.
   Calculate the probability of each of the following events:
   A : « the two drawn bills are of the same category »
   B : « the sum of values of the two drawn bills is 30 000LL ».
2) **One of the two bags S and T is randomly chosen, after which two bills are simultaneously and randomly drawn from this bag.**
   Consider the following events:
   E : « the chosen bag is **S** »
   F : « the sum of values of the two drawn bills is 30 000LL »
   Calculate the probabilities $P(F \cap E)$ and $P(F \cap \overline{E})$. Deduce $P(F)$.
3) **We draw, randomly, one bill from the bag S and one bill from the bag T**.
   Let X be the random variable that is equal to the sum of the values of the two drawn bills.
   a- Verify that $P(X = 60\,000) = \dfrac{3}{64}$.
   b- Determine the probability distribution of X and calculate its mean (expected value).

III– **(3.5 points)**

In the complex plane referred to a direct orthonormal system $(O\,;\vec{u},\vec{v})$, consider the points A and B of affixes 1 and –1 respectively. Let (C) be the circle of center A and of radius 1. The exponential form of the affix z of a point M on (C), other than O, is given by $z = re^{i\theta}$.

Let M' be the point of affix z' such that $z' = \frac{1}{r}e^{i(\pi+\theta)}$.



1) Show that $z' \times \overline{z} = -1$.
2) Show that the points O, M and M´ are collinear.
3) a- Justify that $|\,z-1\,| = 1$.
   b- Prove that $|\,z'+1\,| = |\,z'|$, and deduce that M' moves on a line (d) to be determined.
4) Determine the points M on (C) for which $z' = -\,z$.

IV– **(8 points)**

A- Consider the differential equation (E): $y'' - 4y' + 4y = 4x^2 - 16x + 10$.

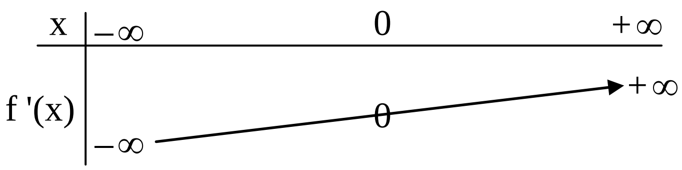
Let $z = y - x^2 + 2x$.

1) Write a differential equation (E') satisfied by z.
2) Solve (E') and deduce the general solution of (E) .
3) Determine the particular solution of (E) whose representative curve, in an orthonormal system, has at the point A(0 ;1) a tangent parallel to the axis of abscissas.

B- Let f be the function that is defined on IR by $f(x) = e^{2x} + x^2 - 2x$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.
   b- Calculate f(1) and f(–1.5) in their decimal forms.
2) The table below is the table of variations of the function f ', the derivative of f.



   a- Determine, according to the values of x , the sign of f '(x) .
   b- Set up the table of variations of f.
3) Draw the curve (C).
4) Let F be the function that is defined on [0 ; + ∞ [by $F(x) = \int_0^x f(t)dt$.
   a- Determine the sense of variations of F.
   b- What is the sign of F(x) ? Justify your answer.

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I	Answers	Marks
1	$\overrightarrow{BA}(2\,;-1\,;-1)$, $\overrightarrow{BC}(0\,;-1\,;1)$; $\overrightarrow{BA}\cdot\overrightarrow{BC}=0$ then ABC is right at B	½
2.a	$x_A+y_A+z_A-4=3+1+0-4=0$; $A\in(P)$. $x_B+y_B+z_B-4=1+2+1-4=0$; $B\in(P)$. $x_C+y_C+z_C-4=1+1+2-4=0$; $C\in(P)$.	½
2.b	$\overrightarrow{N_P}(1;1;1)$ is a director vector of (d) then (d) : $x=\lambda+3$; $y=\lambda+1$; $z=\lambda$. For y = 0 we have λ = –1, then x = 2 and z = – 1, hence $E\in(P)$.	1
3	$\overrightarrow{BE}(1\,;-2\,;-2)$ is normal to (Q) , so (Q) : x – 2y – 2z + r = 0 . $A\in(Q)$ then r = – 1 ; (Q) : x – 2y – 2z – 1 = 0.	½
4.a	$(D)\begin{cases}x+y+z-4=0\\x-2y-2z-1=0\end{cases}$ $(D)\begin{cases}x=3\\y=-t+1\\z=t\end{cases}$ then $\overrightarrow{V_D}(0\,;-1\,;1)=\overrightarrow{BC}$; (BC) // (D). **▶OR :**(BC) is perpendicular to (AB) and it is orthogonal to (EA), so (BC) is perpendicular to plane (EAB) and especially to (EB) , on the other hand since (EB) is perpendicular to (Q) then (BC) is parallel to (Q) . The plane (P), which contains (BC), cuts (Q) along (D) parallel to (BC).	1
4.b	$(BC):\begin{cases}x=1\\y=-m+2\\z=m+1\end{cases}$; M(1 ; – m + 2 ; m +1) ; $d(M;(Q))=\frac{\lvert 1+2m-4-2m-2-1\rvert}{\sqrt{1+4+4}}=2$. **▶OR :** (BC) // (D) and $(D)\subset(Q)$,so (BC) // (Q) . Since $M\in(BC)$; $d(M;(Q))=cst$.	½

<table>
<tr><th>II</th><th>Answers</th><th>Marks</th></tr>
<tr><td>1</td><td>P(A) = P[(10 000 , 10 000) or (20 000 , 20 000)] $=\frac{C_4^2+C_3^2}{C_8^2}=\frac{9}{28}$.
P(B) = P(10 000 , 20 000) = $\frac{C_4^1\times C_3^1}{C_8^2}=\frac{3}{7}$.</td><td>1</td></tr>
<tr><td>2</td><td>$P(F\cap E)=P(E)\times P(F/E)=\frac{1}{2}\times\frac{3}{7}=\frac{3}{14}$.
$P(F\cap\overline{E})=P(E)\times P(F/\overline{E})=\frac{1}{2}\times\frac{C_3^1\times C_5^1}{C_8^2}=\frac{1}{2}\times\frac{15}{28}=\frac{15}{56}$.
P(F) = $P(F\cap E)+P(F\cap\overline{E})=\frac{3}{14}+\frac{15}{56}=\frac{27}{56}$.</td><td>1½</td></tr>
<tr><td>3.a</td><td>P(X = 60 000) = P(50 000 , 10 000) = 1/8 × 3/8 = 3/64.</td><td>½</td></tr>
<tr><td>3.b</td><td><table>
<tr><td>$X=x_i$</td><td>20 000</td><td>30 000</td><td>40 000</td><td>60 000</td><td>70 000</td></tr>
<tr><td>p_i</td><td>$\frac{4}{8}\times\frac{3}{8}=\frac{12}{64}$</td><td>$\frac{4}{8}\times\frac{5}{8}+\frac{3}{8}\times\frac{3}{8}=\frac{29}{64}$</td><td>$\frac{3}{8}\times\frac{5}{8}=\frac{15}{64}$</td><td>$\frac{3}{64}$</td><td>$\frac{1}{8}\times\frac{5}{8}=\frac{5}{64}$</td></tr>
</table>
E(X) = $(24+87+60+18+35)\times\frac{10\,000}{64}=35\,000$.</td><td>1½</td></tr>
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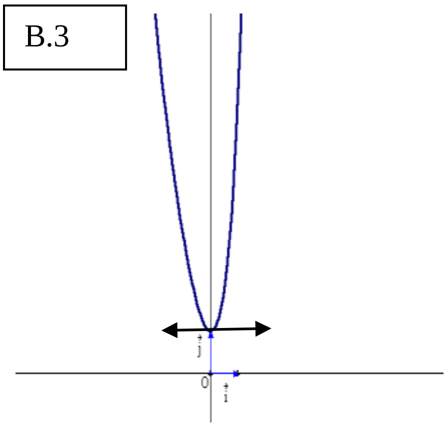
III	Answers	Marks
1	$z'\times\bar{z}=\frac{1}{r}e^{i(\pi+\theta)}\times re^{-i\theta}=\ e^{i\pi}=-1.$	½
2	$(\vec{u}\ ,\overrightarrow{OM})=\theta$, $(\vec{u}\ ,\overrightarrow{OM'})=\pi+\theta$, so $(\overrightarrow{OM},\overrightarrow{OM'})=(\pi+\theta)-\theta=\pi$, hence O , M and M' are collinear. ▶**OR :** $\frac{z'}{z}=-\frac{1}{r^2}$; $z'=-\frac{1}{r^2}z$, so $\overrightarrow{OM'}=-\frac{1}{r^2}\overrightarrow{OM}$ and thus O , M , M' are collinear	½
3.a	$\lvert z-1\rvert=\lvert z_M-z_A\rvert=AM=1$	½
3.b	$\lvert z'+1\rvert=\left\lvert\frac{-1}{\bar z}+1\right\rvert=\left\lvert\frac{\bar z-1}{\bar z}\right\rvert=\frac{\lvert\overline{z-1}\rvert}{\lvert\bar z\rvert}=\frac{\lvert z-1\rvert}{\lvert\bar z\rvert}=\frac{1}{\lvert\bar z\rvert}$ and $\lvert z'\rvert=\frac{\lvert-1\rvert}{\lvert\bar z\rvert}=\frac{1}{\lvert\bar z\rvert}$; $\lvert z'+1\rvert=\lvert z'\rvert$. $\lvert z_{M'}-z_B\rvert=\lvert z_{M'}\rvert$; BM' = OM' ; M' moves on the perpendicular bisector (d) of [OB].	1
4	$z'=-z$; $-z\times\bar z=-1$; $z\times\bar z=1$; $\lvert z\rvert^2=1$; $OM^2=1$; OM = 1 , then M belongs to the circle (C') of center O , radius 1; but M belongs to (C). Then points M are the two points of intersection of (C) and (C'). ▶**OR:** $\lvert -z+1\rvert=\lvert -z\rvert$; $\lvert z-1\rvert=\lvert z\rvert$; AM = OM ; M moves on the perpendicular bisector (D) of [OA].Then points M are the two points of intersection of (D) and (C).	1

IV	Answers	Marks
A.1	$z'=y'-2x+2$ et $z''=y''-2$ $z''+2-4(z'+2x-2)+4(z+x^2-2x)=4x^2-16x+10$ $z''-4z'+4z=0$	½
A.2	$r^2-4r+4=0$; r = 2 double root ; $z=(Ax+B)e^{2x}$ and $y=(Ax+B)e^{2x}+x^2-2x$	1
A.3	y(0) = 1 ; B = 1 y'(0) = 1 with $y'(x)=Ae^{2x}+2(Ax+B)e^{2x}+2x-2$; A + 2B = 2 ; A = 0 ; $y=e^{2x}+x^2-2x$	1
B.1.a	$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}[e^{2x}+x(x-2)]=+\infty$; $\lim_{x\to-\infty}f(x)=\lim_{x\to-\infty}[e^{2x}+x(x-2)]=+\infty$	½
B.1.b	f(1) = 6.39 ; f(–1.5) = 5.30 .	½
B.2.a	f '(x) < 0 for x < 0 ; f '(x) > 0 for x > 0.	1
B.2.b	$\begin{array}{c\|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & +\infty & \searrow & 1 & \nearrow & +\infty \end{array}$	½
B.3		1
B.4.a	F'(x) = f(x) > 0 for x ≥ 0 (min (f(x)) =1) , then F is strictly increasing over [0 ; + ∞ [.	1
B.4.b	f(t) > 0 and x ≥ 0 then $\int_0^x f(t)dt\ge0$, so F(x) ≥ 0. ▶**OR :** F is increasing and F(0) = 0 , then F(x) ≥ 0.	1