امتحانات الشهادة الثانوية العامة
فرع العلوم العامة

دورة سنة ٢٠٠٧ العادية

مسابقة في مادة الفيزياء
المدة: ثلاث ساعات

الاسم:
الرقم:

This exam is formed of four exercises in four pages numbered from 1 to 4
The use of a non-programmable calculator is recommended

First exercise *(7 ½ pts)* **Solid in rotation**

Consider a rigid rod AB, of negligible mass and of length AB = L = 80 cm. The rod may rotate around a horizontal axis (Δ), perpendicular to it through its midpoint O. Two identical particles, each of mass m = 10g , may slide along this rod. Take $g = 10\ m/s^2$; $0.32\pi = 1$.

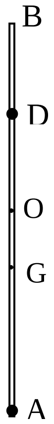

Fig.1

I- Work done by the couple of friction

We fix one of the two particles at the end A of the rod while the other particle is fixed at another point D , at a distance $\frac{L}{4}$ from O .

G being the centre of gravity of the system (S) formed of the rod and the two particles, we suppose OG = a .

Take as a gravitational potential energy reference, the horizontal plane through G when (S) is in the position of stable equilibrium (Fig.1).

1) Show that $a = \frac{L}{8}$.

2) (S) is in its stable equilibrium position .At the instant $t_0 = 0$, we communicate to (S) an initial kinetic energy $E_0 = 1.95 \times 10^{-4}$ J ; (S) oscillates then around (Δ), on both sides of its position of stable equilibrium. At an instant t, OG makes an angle θ with the vertical through O.

a) Neglecting friction, show that:

i. the expression of the gravitational potential energy of the system [(S),Earth] is $P.Eg = 2mga(1-\cos\theta)$;

ii. the value of the mechanical energy of the system [(S), Earth] is E_0 ;

iii. the value of the angular amplitude of the motion of (S) is $\theta_m = 8^o$.

b) In reality, the forces of friction form a couple whose moment about the axis (Δ) is $\mathcal{M}$. We suppose that $\mathcal{M}$ is constant .The measurement of the first maximum elongation of (S) is then $\theta_{1m} = 7°$ at the instant t_1.

i. Determine the expression giving the variation of the mechanical energy of the system [(S), Earth] between t_0 and t_1 in terms of m, g, a, θ_{1m} and E_0 .

ii. Deduce the value W of the work done by $\mathcal{M}$ between t_0 and t_1.

II- Moment of the couple of friction

We fix each particle on an extremity of the rod (figure 2). At the instant $t_0= 0$, and we give (S), a rotational speed $N_0 = 1$ turn/s and we suppose that $\mathcal{M}$ keeps the same preceding value.

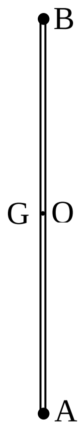

Fig.2

1) Show that the moment of inertia of (S) with respect to (Δ) is $I = 32 \times 10^{-4}\ kg.m^2$.

2) Show that the value of the angular momentum of (S) with respect to (Δ), at t_0, is $\sigma_0 = 2 \times 10^{-2}\ kg.m^2/s$.

3) a) Give the names of the external forces acting on (S).

b) Show that the value of the resultant moment of these forces, with respect to (Δ), is equal to $\mathcal{M}$.

c) Find, applying the theorem of angular momentum, the expression of the angular momentum σ of (S) with respect to (Δ), in terms of $\mathcal{M}$, t and σ_0 .

4) Launched with the rotational speed $N_0 = 1$ turn/s, (S) stops at the instant t' = 52.8 s. Determine then the value of $\mathcal{M}$.

III- Relation between W and $\mathcal{M}$

Referring to the parts **I** and **II**, verify that the work W is $W = \mathcal{M} \times \theta_{1m}$.

Second exercise **(6 ½ pts) Energy dissipated during the charging of a capacitor**

The object of this exercise is to determine the energy dissipated, by Joule's effect, during the charging of a capacitor.

We charge a capacitor of capacitance $C = 5 \times 10^{-3}$F, initially neutral, using an ideal generator of constant voltage of e.m.f E through a resistor of resistance R = 200 Ω (fig.1).

At the instant $t_0 = 0$, the switch K is closed. The circuit thus carries a current i at the instant t.

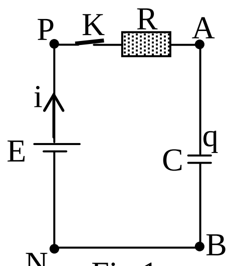

Fig.1

I-Exploiting a waveform

Using an oscilloscope, we display the variations of the voltage $u_R = u_{PA}$ across the resistor and that of $u_C = u_{AB}$ across the capacitor.

We obtain the waveforms of figure 2.

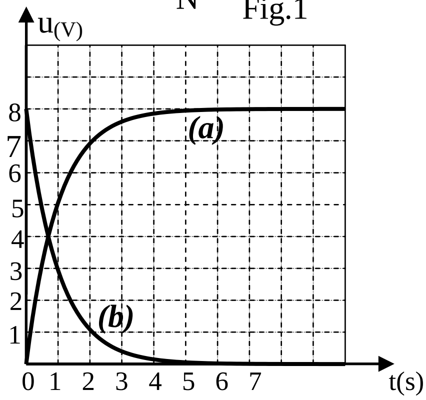

Fig.2

1) The curve (b) represents the variation of u_R as a function of time. Why?

2) Determine, using the waveforms:

a) the value of E ;

b) the maximum value I of i;

c) the time constant τ of the RC circuit.

3) Give the time at the end of which the capacitor will be practically completely charged.

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II- Theoretical study of charging

1) Show that the differential equation in u_C may be written as: $E = RC\frac{du_C}{dt} + u_C$

2) The solution of this equation has the form $u_C = A\,e^{\frac{-t}{\tau}} + B$ where A, B and τ are constants.

a) Determine , starting from the differential equation ,the expression of B in terms of E and that of τ in terms of R and C.

b) Using the initial condition, determine the expression of A in terms E.

3) Show that: $i = \frac{E}{R}\,e^{\frac{-t}{\tau}}$.

III- Energetic study of charging

1) Calculate the value of the electric energy W_C stored in the capacitor at the end of the charging process.

2) The instantaneous electric power delivered by the generator at the instant t is $p = \frac{dW}{dt} = Ei$ where W is the electric energy delivered by the generator between the instants t_0 and t.

a) Show that the value of the electric energy delivered by the generator during the whole duration of charging is 0.32 J.

b) Deduce the energy dissipated due to Joule's effect in the resistor.

Third exercise **(6 ½ pts) Ionization energy**

Given: $1\text{ eV} = 1.6\times10^{-19}$J ; Planck's constant $h = 6.62 \times 10^{-34}$ J.s; speed of light in vacuum $c = 3\times10^{8}$ m/s.

The object of this exercise is to compare the ionization energy of the hydrogen atom with that of the helium ion He^{+} and that of the lithium ion Li^{2+} <u>each having only one electron in the outermost shell</u>.

The quantized energy levels of each is given by the expression $E_n = -\frac{E_0}{n^2}$ where E_0 is the ionization energy and n is a non-zero positive whole number.

I- Interpretation of the existence of spectral lines

1) Due to what is the presence of emission spectral lines of an atom or an ion?

2) Explain briefly the term "quantized energy levels"**.**

3) Is a transition from an energy level m to another energy level p (p < m) a result of an absorption or an emission of a photon? Why?

II- Atomic spectrum of hydrogen

For the hydrogen atom $E_0 = 13.6$ eV.

1) A hydrogen atom, found in its ground state, interacts with a photon of energy 14 eV.

a) Why?

b) A particle is thus liberated. Give the name of this particle and calculate its kinetic energy.

2) ***a)*** Show that the expression of the wavelengths λ of the radiations emitted by the hydrogen atom is:

$\frac{1}{\lambda} = R_1(\frac{1}{p^2} - \frac{1}{m^2})$ where m and p are two positive whole numbers so that m > p and R_1 is a positive constant to be determined in terms of E_0, h and c. .

b) Verify that $R_1 = 1.096 \times 10^7$ m^{-1}.

III- Atomic spectrum of the helium ion He^+

The spectrum of the ion He^+ is formed, in addition to others, of two lines whose corresponding reciprocal wavelengths $\frac{1}{\lambda}$ are: 3.292×10^7 m^{-1}; 3.901×10^7 m^{-1} respectively. These lines correspond, respectively, to the transitions: ($m = 2 \rightarrow p = 1$) and ($m = 3 \rightarrow p = 1$).

1)a) Show that the values of $\frac{1}{\lambda}$ satisfy the relation $\frac{1}{\lambda} = R_2(\frac{1}{p^2} - \frac{1}{m^2})$ where R_2 is a positive constant.

b) Deduce that $R_2 = 4.389 \times 10^7$ m^{-1}.

2) Find a relation between R_2 and R_1.

IV-Atomic spectrum of the lithium ion Li^{2+}

Also, the ion Li^{2+} may emit radiations whose wavelengths λ are given by : $\frac{1}{\lambda} = R_3(\frac{1}{p^2} - \frac{1}{m^2})$

where m and p are two positive whole numbers so that m > p and $R_3 = 9.860 \times 10^7$ m^{-1} .

Find a relation between R_3 and R_1.

V-Charge number and ionization energy

The charge numbers Z of the elements hydrogen, helium and lithium are respectively 1 , 2 and 3 .

Compare the ionization energy of the hydrogen atom with that of He^+ ion and that of Li^{2+} ion. Conclude.

<u>Fourth exercise</u> (7 pts) An analogy

The object of this exercise is to show evidence of the analogy between a mechanical oscillator and an electric oscillator in the case of free oscillations.

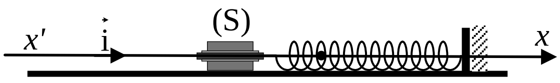

Fig.1

A- Mechanical oscillator

A horizontal mechanical oscillator is formed of a solid (S) of mass m = 0.546 kg and a spring of un-jointed turns of stiffness k = 5.70 N/m and of negligible mass .

The center of mass G of (S) is initially at the equilibrium position O on the axis x'x.

(S), shifted from O by a certain distance, is then released without initial velocity at the instant $t_0 = 0$.

G thus performs a rectilinear motion along the axis x'x (fig.1). At the instant t, its abscissa is x ($\overrightarrow{OG} = x\,\vec{i}$) and its velocity is $\vec{V}$ ($\vec{V} = V\vec{i} = \frac{dx}{dt}\vec{i}$) .

The horizontal plane through the axis x'x is taken as a gravitational potential energy reference.

I – General study

1) Write down the expression of the mechanical energy M.E of the system [oscillator, Earth] in terms of m, k, x and V.

2) Determine the expression giving $\frac{d(M.E)}{dt}$, the derivative of M.E with respect to time.

II- Free non-damped oscillations

We neglect all friction.

1) Derive the second order differential equation that governs the variations of x as a function of time.

2) Deduce the expression of the proper frequency f_0 of the oscillator and show that its value is 0.51 Hz.

III- Free damped oscillations

In reality, the force $\vec{F}$ of friction is not negligible and its expression is given by: $\vec{F} = -\lambda\ \vec{V}$ at an instant t , λ being a positive constant.

1) Derive the second order differential equation describing the variations of x as a function of time knowing that $\frac{d(M.E)}{dt} = \vec{F}.\vec{V}$

2) The adjacent figure2 shows the variations of x as a function of time.

a) How does the effect of the force of friction appear?

b) Determine the pseudo-frequency f of the mechanical oscillations.

c) Calculate the value of λ, knowing that f is given by the expression :

$$f^2 = (f_0)^2 - \frac{1}{4\pi^2}\left(\frac{\lambda}{2m}\right)^2 .$$

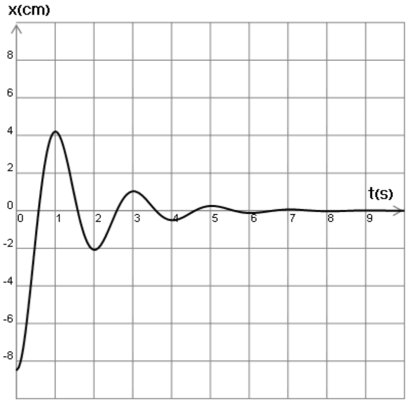

Fig.2

B-Electric oscillator

This oscillator is a series circuit formed of a coil of inductance L= 43 mH and of resistance r =11 Ω, a resistor of adjustable resistance R , a switch K and a capacitor of capacitance C = 4.7 μF initially charged with a charge Q (Fig.3).

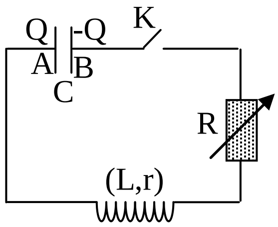

Fig. 3

We close the switch K at the instant $t_0 = 0$. The circuit is thus the seat of electric oscillations. At the instant t, the armature A carries a charge q and the circuit carries a current i (Fig.4).

1) Write down the expression of the electromagnetic energy E of the circuit at the instant t (total energy of the circuit) as a function of L, i, q and C.

2) Knowing that $\frac{dE}{dt} = -(R+r)i^2$, derive the second order differential equation of the variations of q as a function of time.

3) Give the expression of the proper frequency f'_0 of the electric oscillations and show that its value is 354.2 Hz.

4) The figure 5 gives the variations of q as a function of time.

a) Due to what is the decrease with time in the amplitude of oscillations?

b) Determine the pseudo-frequency f ′ of the electric oscillations.

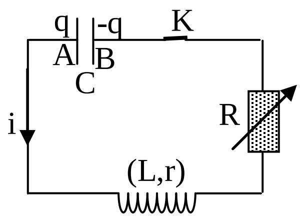

Fig. 4

C-An analogy

1) Match each of the physical mechanical quantities x, V, m, λ and k with its corresponding convenient electric quantity.

2) a) Deduce the relation between f ′, f'_0, L and (R + r).

b) Calculate the value of R.

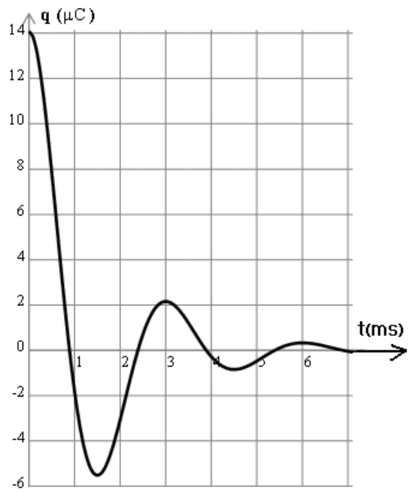

Fig.5

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First exercise (7 ½ pts)

I- 1) **a = OG =** $\dfrac{m\frac{L}{2} - m\frac{L}{4}}{2m} = \dfrac{L}{8}$. **(1/2 pt)**

2) a- i) $PE_g = M_t\, gh_G = 2mg\,(a - a\cos\theta) = 2mga(1-\cos\theta)$. **(1/2pt)**

ii) The mechanical energy is conserved because friction is neglected
$\Rightarrow ME_i = ME_f \Rightarrow ME = KE_0 + PE_{g0} = KE_0 + 0$ (For $\theta = 0$).**(1/2pt)**

iii) $ME_i = ME_f \Rightarrow 1.95\times10^{-4} = 2mg.a\,(1 - \cos\theta_m) \Rightarrow \theta_m = 8^0$ **(1/2pt)**

b- i) $\Delta ME = 2mga(1 - \cos\theta_{1m}) - KE_0$ **(1/2pt)**

ii) $W = \Delta ME = 2 \times 0.01 \times 10 \times 0.1(1 - 0.99255) - 1.95 \times 10^{-4}$
$= 1.49 \times 10^{-4} - 1.95 \times 10^{-4} = -4.6 \times 10^{-5}$ J. **(1/2 pt)**

II- 1) $I = 2m\dfrac{L^2}{4} = 32 \times 10^{-4}$ kg.m². **(1/2 pt)**

2) $\sigma_0 = I\,\theta_0' = I\times2\pi N_0 = 2\times10^{-2}$ kg.m²/s. **(3/4pt)**

3) a) The forces applied on (S) are : weight $2\,m\vec{g}$
the reaction $\vec{R}$ of axis (Δ) and the couple of friction. **(1/2pt)**

b) $\Sigma\,\mathcal{M}/\Delta = \mathcal{M}(\vec{R})/\Delta + M(2\,m\vec{g})/\Delta + \mathcal{M}$ (couple)/Δ;
or $\mathcal{M}(\vec{R}) = \mathcal{M}$(weight) = 0 (because the 2 forces passes through the axis) ;
$\Rightarrow \Sigma\,\mathcal{M} = \mathcal{M}$ **(1/2pt)**

c) $\dfrac{d\sigma}{dt} = \Sigma\,\mathcal{M} = \mathcal{M} \Rightarrow \sigma = \mathcal{M}\,t + \sigma_0$. **(1 pt)**

4) $\theta' = 0 \Rightarrow \sigma = 0 = \mathcal{M}\,t' + \sigma_0 \Rightarrow \mathcal{M} = -\dfrac{\sigma_0}{t'} = -3.78\times10^{-4}$ m.N. **(3/4 pt)**

III- $\mathcal{M}\,\theta = -3.78\times10^{-4}\times\dfrac{7\times\pi}{180} = -4.6\times10^{-5}$ J and $W = -4.6\times10^{-5}$ J
$\Rightarrow W = \mathcal{M}\,\theta$ (θ in rad). **(1/2pt)**

Second exercise (6 ½ pts)

I- 1) The current i decreases with time, [at the end of charging i = 0] ⇒ the voltage $u_R = Ri$ is represented by the curve (b). **(1/2 pt)**

2) a) Explantion : at the end of charging $u_C = E$; E = 8 V. **(1/2 pt)**

b) $RI = 8 \Rightarrow I = \dfrac{8}{200} = 0.04$ A. **(1/2 pt)**

c) Method **(1/2 pt)** τ = 1s. **(1/4 pt)**

3) 5 τ = 5 s **(1/4 pt)**

II- 1) $u_R = Ri = R\dfrac{dq}{dt} = RC\dfrac{du_C}{dt}$; thus $E = u_R + u_C = RC\dfrac{du_C}{dt} + u_C$ **(1/2 pt)**

2) a) $u_C = A\,e^{-\frac{t}{\tau}} + B \Rightarrow (-\dfrac{RCA}{\tau})\,e^{-\frac{t}{\tau}} + A\,e^{-\frac{t}{\tau}} + B = E \Rightarrow B = E$ and $\tau = RC$ **(1 pt)**

b) For t = 0 $u_C = 0 = A + B \Rightarrow A = -B = -E$. **(1/2 pt)**

3) $u_C = E(1 - e^{-\frac{t}{\tau}})$ thus $i = C\dfrac{du_C}{dt} = C\dfrac{E}{RC}\,e^{-\frac{t}{\tau}} = \dfrac{E}{R}\,e^{-\frac{t}{\tau}}$. **(1/2 pt)**

III- 1) $W_C = \dfrac{1}{2}CE^2 = 0.16$ J **(1/2 pt)**

2) a) $\dfrac{dW}{dt} = Ei \Rightarrow W$ = primitive of Ei = primitive of $E\dfrac{E}{R}\,e^{-\frac{t}{\tau}} \Rightarrow$

$W = -CE^2\,e^{\frac{-t}{\tau}} + cte.$
For t = 0, the electric energy delivered by the generator is zero ⇒
cte = CE² ⇒ the expression of the dissipated energy as a function of time is :

$W = CE^2(1 - e^{\frac{-t}{\tau}})$.

For t = 5RC (as $t \to \infty$), $1 - e^{\frac{-t}{\tau}} \to 1$ and $W = CE^2 = 0.32$ J **(3/4 pt)**

b) $W_R = W_e - W_C = CE^2 - \dfrac{1}{2}CE^2 = \dfrac{1}{2}CE^2 = 0.16$ J **(1/4 pt)**

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Third exercise (6 ½ pts)

I -

1) The presence of the lines in this emission spectrum is due to photon, the wavelength is a well determined value that the atom emits it when it undergoes a down ward transition from a higher energy level to a lower energy level. **(1/2 pt)**

2) The atom absorbed a well determined value. **(1/2 pt)**

3) $E_P < E_m \Rightarrow$ the atom loses energy by emitting one photon. **(1/2 pt)**

II – 1) a) The energy of the photon (14 eV) greater than the ionization energy (13.6 eV) **(1/4pt)**

b) Electron ; KE = 14 – 13.6 = 0.4 eV. **(1/2 pt)**

2) a) When an atom of the hydrogenoied pass from a level m to a lower level p, it emits a photon of energy $h\nu = \frac{hc}{\lambda} = E_m - E_p = -\frac{E_0}{m^2} + \frac{E_0}{p^2} \Rightarrow$

$\frac{1}{\lambda} = \frac{E_0}{hc}\left(\frac{1}{p^2} - \frac{1}{m^2}\right)$ it has the form of $\frac{1}{\lambda} = R_1\left(\frac{1}{p^2} - \frac{1}{m^2}\right)$ with $R_1 = \frac{E_0}{hc}$ **(1 1/4 pt)**

b) $R_1 = \frac{E_0}{hc} = \frac{13.6\times1.6\times10^{-19}}{6.62\times10^{-34}\times3\times10^{8}} = 1.096\times10^{7}\ m^{-1}$. **(1/2 pt)**

III - 1) a) We get : $R_2 = \frac{1}{\lambda\left(\frac{1}{p^2} - \frac{1}{m^2}\right)}$

For p = 1 and m = 2 gives $\frac{3.292\times10^{7}}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} = 4.389\times10^{7}\ m^{-1}$

For p = 1 and m = 3 gives $\frac{3.901\times10^{7}}{\left(\frac{1}{1^2} - \frac{1}{3^2}\right)} = 4.389\times10^{7}\ m^{-1}$

The value of $\frac{1}{\lambda\left(\frac{1}{p^2} - \frac{1}{m^2}\right)}$ is the same for the two transitions. **(1pt)**

b) The calculation gives $R_2 = 4.389\times10^{7}\ m^{-1}$. **(1/4 pt)**

2) $\frac{R_2}{R_1} = 4$ **(1/4 pt)**

IV - $\frac{R_3}{R_1} = 9$. **(1/4 pt)**

V - As Z increases, R increases because $R = \frac{E_0}{hc} \Rightarrow$ the ionization energy E_0 increases as Z increases. **(3/4pt)**

Fourth exercise (7pts)

A- I- 1) $ME = ½ mV^2 + ½ kx^2$ **(1/4 pt)**

2) $\frac{dME}{dt} = mx'x'' + kxx'$ **(1/4 pt)**

II- 1) in this case $\frac{dME}{dt} = 0 \Rightarrow x'' + \frac{k}{m}x = 0$ **(1/4 pt)**

2) The proper angular frequency of oscillations is $\omega_0 = \sqrt{\frac{k}{m}} \Rightarrow$ the proper frequency is

$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. **(1/2 pt)**

$f_0 = 0.51$ Hz. **(1/4 pt)**

III- 1) $\frac{dME}{dt} = \vec{F}\cdot\vec{V} \Rightarrow mx'x'' + kxx' = -\lambda x'x' \Rightarrow x'' + \frac{\lambda}{m}x' + \frac{k}{m}x = 0$. **(1/2 pt)**

2) a) The effect of the force of friction is to decrease the amplitude **(1/4 pt)**

b) The pseudo-period is T = 2 s $\Rightarrow$ f = 0.5 Hz. **(1/2 pt)**

c) $\lambda = 0.685$ kg/s. **(1/2 pt)**

B- 1) $E = ½ Li^2 + ½ \frac{q^2}{C}$. **(1/4 pt)**

2) $\frac{dE}{dt} = -(R+r)i^2 \Rightarrow Lii' + \frac{1}{C}qq'$; with i = -q' and i' = -q''

$\Rightarrow Lq'q'' + \frac{1}{C}qq' = -(R+r)(q')^2 \Rightarrow q'' + \frac{(R+r)}{L}q' + \frac{1}{LC}q = 0$. **(1/2 pt)**

3) $f'_0 = \frac{1}{2\pi\sqrt{LC}}$. $f'_0 = 354.2$ Hz. **(1/2 pt)**

4) a) the energy lost in the circuit is due to Joule's effect. **(1/4 pt)**

b) T = 3 ms $\Rightarrow$ f' = 333.3 Hz. **(1/2 pt)**

C –

1) x ---------> q **(1/4 pt)**

V----------> i **(1/4 pt)**

m ---------> L **(1/4 pt)**

λ ---------> (R+r) **(1/4 pt)**

k ---------> $\frac{1}{C}$ **(1/4 pt)**

2) a) $f'^2 = (f'_0)^2 - \frac{1}{4\pi^2}\left(\frac{R+r}{2L}\right)^2$ **(1/4 pt)**

b) R = 54 Ω. **(1/4 pt)**