This test includes three mandatory exercises. The use of non-programmable calculators is allowed.

**Exercise 1 (6½ points)  
Young’s slits**

Consider the Young’s slits device (Doc 1) made up of two very thin and horizontal slits $S_1$ and $S_2$ separated by a distance $a = 1 \text{ mm}$, a screen (E) parallel to the plane containing $S_1$ and $S_2$ and a monochromatic light source S.

The screen (E) is at a distance $D = 2 \text{ m}$ from the midpoint I of $[S_1S_2]$.

The light source (S) is on the perpendicular bisector of $[S_1S_2]$. This bisector meets the screen (E) at a point O.

The wavelength in air of the monochromatic light is $\lambda = 650 \text{ nm}$.

1) A pattern is observed on the screen (E). Indicate the name of the correspondent phenomenon.
2) State the conditions ensured by $S_1$ and $S_2$ in order to obtain this pattern.
3) Consider a point M of the pattern observed on the screen (E) such as $OM = x$. Take $d_1 = S_1M$ and $d_2 = S_2M$. Write the relation that gives the optical path difference $\delta = d_2 - d_1$ at M in terms of $a$, $D$ and $x$.
4) Define the interfringe distance $i$.
5) Give the expression of $i$ in terms of $\lambda$, $D$ and $a$, then calculate its value.
6) The point O coincides with the centre of a fringe called central fringe.
   6-1) Calculate the optical path difference $\delta$ at O.
   6-2) Specify whether this fringe is bright or dark.
7) Let N be the centre of a fringe where $\delta = 2.275 \text{ mm}$. Specify whether this fringe is bright or dark.
8) $S$ is at a distance $d = 10 \text{ cm}$ from I. We displace $S$ vertically of a distance $y = 1 \text{ cm}$ to the side of $S_1$. The new optical path difference is then: $\delta' = \frac{ax}{D} + \frac{ay}{d}$. Specify the direction of the displacement of the centre of the central fringe (to the side of $S_1$ or $S_2$) and calculate the displacement.

**Exercise 2 (6½ points)  
(RC) series circuit**

The electric circuit of the document (Doc 2) is formed of:
- A generator delivering across its terminals a constant voltage $E = 8 \text{ V}$;
- A resistor of unknown resistance $R$;
- A capacitor of capacitance $C = 100 \mu\text{F}$, initially discharged;
- A switch $K$.

![Diagram](Doc 2)
At the instant \( t_0 = 0 \), we close the switch K.
At an instant \( t \), the capacitor is charged by \( q \) and the circuit carries a current \( i \).
1) Redraw the figure of the document (Doc 2) and show the connections of an oscilloscope that allows to display the voltage \( u_G = E \) across the generator and the voltage \( u_C = u_{AB} \) across the capacitor.
2) Write the expression of the current \( i \) in terms of \( q \).
3) Deduce the expression of \( i \) in terms of the capacitance \( C \) and the voltage \( u_C \).
4) Determine the differential equation that describes the variation of \( u_C \) as a function of time.
5) The solution of this differential equation is: \( u_C = D \left( 1 - e^{-\frac{t}{\tau}} \right) \). Determine the expressions of the constants \( D \) and \( \tau \) in terms of \( E \), \( R \) and \( C \).
6) Determine, at the instant \( t = \tau \), the expression of the voltage \( u_C \) in terms of \( E \).
7) Referring to the graph of \( u_C = f(t) \) of the document (Doc 3) below:
7-1) Determine the value of \( \tau \).
7-2) Deduce the value of the resistance \( R \).

![Graph](Doc 3)

8) Determine the expression of the current \( i \) as a function of time \( t \).
9) Deduce the value of the current \( i \) in steady state.

**Exercise 3 (7 points)  Horizontal elastic pendulum**

An air puck \( (S) \) of mass \( m = 709 \) g is attached to the free end of a spring \( (R) \) of un-jointed turns, of negligible mass and of stiffness \( k = 7 \) N.m\(^{-1}\).
This puck, of centre of mass \( G \), may slide without friction on a horizontal rail (Doc 4). The document (Doc 4) shows a horizontal axis \( Ox \) of origin \( O \). At equilibrium, \( G \) coincides with \( O \).
(S) is shifted 3 cm from \( O \) \((\overrightarrow{OG_0} = x_0\overrightarrow{i} = 3 \overrightarrow{i})\) in the positive direction and released without velocity at the instant \( t_0 = 0 \).
At an instant \( t \), \( x \) is the abscissa of \( G \) and \( v = \frac{dx}{dt} \) is the algebraic measure of its velocity.

![Diagram](Doc 4)
1) The mechanical energy of the system ((S), (R), Earth) is conserved.

1-1) Determine the second order differential equation in x.

1-2) Verify that \( x = x_m \cos \left( \sqrt{\frac{k}{m}} t + \phi \right) \) is the solution of this differential equation.

1-3) Calculate the values of the constants \( x_m \) and \( \phi \).

2) Write down the expression of the natural period \( T_0 \) of the motion in terms of \( k \) and \( m \) then calculate its value.

3) The document (Doc 5) below shows the curves giving the variations of the kinetic energy \( KE \) of (S), of the elastic potential energy \( PE_e \) of (R) and of the mechanical energy \( ME \) of the system ((S), (R), Earth). Identify the curves \( KE \), \( PE_e \) and \( ME \) of the document (Doc 5).

4) Each of the curves A and C is sinusoidal of a period T. Referring to the graph of document (Doc 5):

4-1) Pick up the value of the period T;

4-2) Compare its value to the natural period \( T_0 \) of the motion.
### Exercise 1 (6½ points) Young’s slits

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Interference.</td>
<td>¼</td>
</tr>
<tr>
<td>2</td>
<td>The light sources must be synchronous (they must have the same frequency) and coherent (they must keep a constant phase difference).</td>
<td>¼ ¼</td>
</tr>
<tr>
<td>3</td>
<td>( \delta = \frac{ax}{D} )</td>
<td>½</td>
</tr>
<tr>
<td>4</td>
<td>The interfringe distance is the distance between the centers of two consecutive fringes of the same nature.</td>
<td>½</td>
</tr>
<tr>
<td>5</td>
<td>( i = \frac{\lambda D}{a} ) [ \Rightarrow i = \frac{650 \times 10^{-9} \times 2}{10^{-3}} \Rightarrow i = 1.3 \times 10^{-3} \text{ m} ]</td>
<td>¼ ½</td>
</tr>
<tr>
<td>6-1</td>
<td>( d_2 = d_1 ) [ \Rightarrow \delta = d_2 - d_1 = 0 ] or ( x = 0 ) [ \Rightarrow \delta = \frac{ax}{D} = 0 ]</td>
<td>¼ ¼ ¼</td>
</tr>
<tr>
<td>6-2</td>
<td>( \delta = 0 ) so ( \delta = k \lambda ) with ( k = 0 \in \mathbb{Z} ) [ \text{The interference is constructive and the fringe is bright.} ]</td>
<td>¼ ¼ ¼ ¼</td>
</tr>
<tr>
<td>7</td>
<td>( \delta = \frac{2.275 \times 10^{-6}}{650 \times 10^{-9}} = 3.5 ) [ \Rightarrow \frac{\delta}{\lambda} = k + \frac{1}{2} \text{ with } k = 1 \in \mathbb{Z} ] [ \text{The interference is destructive and the fringe is dark.} ]</td>
<td>¼</td>
</tr>
<tr>
<td>8</td>
<td>( \delta = \frac{ax_o}{D} + \frac{ay}{d} = 0 ) [ \Rightarrow x_o = -\frac{y \cdot D}{d} ] [ \Rightarrow x_o = -\frac{10^{-2} \times 2}{10 \times 10^{-2}} = -0.2 \text{ m} ] [ \text{The central fringe moves } 0.2 \text{ m towards } S_2 ]</td>
<td>¼ ¼ ¼ ¼</td>
</tr>
</tbody>
</table>
Exercise 2 (6½ points)  (RC) series circuit

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<tr>
<td>1</td>
<td><img src="image" alt="Diagram" /></td>
<td>½</td>
</tr>
<tr>
<td>2</td>
<td>$i = \frac{dq}{dt}$</td>
<td>½</td>
</tr>
<tr>
<td>3</td>
<td>$q = Cu_C \text{ so } i = C\frac{du_C}{dt}$</td>
<td>½</td>
</tr>
</tbody>
</table>
| 4        | Law of addition of voltages:  
$u_{PM} = u_{PA} + u_{AB} + u_{BM}$  
$u_{PA} = u_R; u_{AB} = u_C$ and $u_{BM} = 0$  
So: $u_R + u_C = E \ \forall t$  
Ohm’s law: $u_R = Ri \Rightarrow u_R = RC\frac{du_C}{dt}$  
The differential equation in terms of $u_C$ is then: $RC\frac{du_C}{dt} + u_C = E$ | ½    |
| 5        | $u_C = D\left(1-e^{-\frac{t}{\tau}}\right) \Rightarrow u_C = D - De^{-\frac{t}{\tau}}$  
$\frac{du_C}{dt} = -D\left(-1\right)e^{-\frac{t}{\tau}} = D\frac{1}{\tau}e^{-\frac{t}{\tau}}$  
Replace $u_C$ and $\frac{du_C}{dt}$ by their expressions in the differential equation.  
We get:  
$RC\frac{D}{\tau}e^{-\frac{t}{\tau}} + D - De^{-\frac{t}{\tau}} = E \ \forall t$  
$D\left(\frac{RC}{\tau} - 1\right)e^{-\frac{t}{\tau}} + D - E = 0 \ \forall t$  
Identifying, we get:  
$D-E=0 \Rightarrow D=E$  
$\left(\frac{RC}{\tau} - 1\right) = 0 \Rightarrow \tau = RC$ | ½    |
| 6        | At $t = \tau$; $u_C = E\left(1-e^{-\frac{\tau}{\tau}}\right) = E\left(1-e^{-1}\right) \approx 0.63E$ | ½    |
| 7-1      | At $t = \tau$; $u_C = 0.63E = 0.63 \times 8 = 5.04 \text{ V} \approx 5 \text{ V}$  
from the graph we get: $\tau = 2 \text{ s}$ | ½    |
| 7-2      | $R = \frac{\tau}{C} \Rightarrow R = \frac{2}{100\times10^{-6}} = 2 \times 10^4 \Omega$ | ½    |
Exercise 3 (7 points)  

**Horizontal elastic pendulum**

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<tr>
<td>1-1</td>
<td>PE(_g) = constant because the rail is horizontal (\Rightarrow) (\frac{dPE_g}{dt} = 0)</td>
<td>(\frac{1}{2})</td>
</tr>
</tbody>
</table>
| | ME = KE + PE\(_e\) + PE\(_g\)  
The mechanical energy of the system (puck, spring, Earth) is conserved  
ME = \(\frac{1}{2}mv^2 + \frac{1}{2}kx^2 + PE_g\) = constant \(\forall t\) \(\Rightarrow\) \(\frac{d(ME)}{dt} = 0\) \(\forall t\)  
\(\Rightarrow\) mx'' + kx' + 0 = 0 \(\forall t\) \(\Rightarrow\) mx\(\left(x'' + \frac{k}{m}x\right) = 0\) \(\forall t\)  
The product of the two quantities is always nil. But mx' is not always nil, we get: \(x'' + \frac{k}{m}x = 0\) \(\forall t\) | \(\frac{1}{2}\) |
| 1-2 | \(x = x_m \cos \left(\frac{k}{m}t + \varphi\right)\) \(\Rightarrow\)  
\(x' = -x_m \sqrt{\frac{k}{m}} \sin \left(\frac{k}{m}t + \varphi\right)\)  
\(\Rightarrow\) \(x'' = -\frac{k}{m}x_m \cos \left(\frac{k}{m}t + \varphi\right) = -\frac{k}{m}x\) | \(\frac{1}{2}\) |
| 1-3 | At \(t_0 = 0\) s; \(v_0 = x'_0 = -x_m \sqrt{\frac{k}{m}} \sin \varphi = 0\) \(\Rightarrow\) \(\sin \varphi = 0\) \(\Rightarrow\) \(\varphi = 0\) or \(\varphi = \pi\) rd  
At \(t = 0\) s; \(x_0 = x_m \cos \varphi > 0\)  
For \(\varphi = 0\) rd: \(x_0 = x_m = +3\) cm (acceptable because \(x_m > 0\))  
For \(\varphi = \pi\) rd: \(x_0 = -x_m = +3\) cm \(\Rightarrow\) \(x_m = -3\) cm (rejected because \(x_m\) is always positive) | \(\frac{1}{2}\) |
| 2 | \(T_0 = 2\pi \sqrt{\frac{m}{k}}\) \(\Rightarrow\) \(T_0 = 2\pi \sqrt{\frac{0.709}{7}} = 2\) s | \(\frac{1}{2}\) |
| 3 | The curve A corresponds to PE\(_e\) because at \(t_0 = 0\) s, \(x_0 \neq 0\) but \(PE_e = \frac{1}{2}kx^2\) so \(PE_e(0) \neq 0\) J  
The curve B corresponds to ME because it has a constant value  
The curve C corresponds to KE because at \(t = 0\) s, \(v = 0\) m/s but \(KE = \frac{1}{2}mv^2\) so \(KE(0) = 0\) J | \(\frac{1}{2}\) |
| 4-1 | From the graph we get: \(T = 1\) s | \(\frac{1}{4}\) |
| 4-2 | \(T = T_0/2\) | \(\frac{1}{4}\) |