

دورة سنة ٢٠٠٨ العادية	امتحانات الشهادة الثانوية العامة فرع العلوم العامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ثلاث ساعات	

This exam is formed of four exercises in four pages numbered from 1 to 4.

The use of non-programmable calculator is recommended

First exercise (7.5 points) Compound pendulum

A compound pendulum is formed of a rod AB of negligible mass, which can rotate without friction in a vertical plane around a horizontal axis (Δ) passing through a point O of the rod so that $OB = d$. A particle of mass M is fixed at point B and another particle C of mass $m < M$, which can slide on the part OA of the rod is placed at a distance $OC = x$ of adjustable value. Let $a = OG$ be the distance between O and the center of gravity G of the pendulum (Fig.1). The gravitational potential energy reference is the horizontal plane containing O.

$$g = 10\text{m/s}^2 ; \pi^2 = 10 ; \sin \theta = \theta \text{ and } \cos \theta = 1 - \frac{\theta^2}{2}, (\theta \text{ in rad}) \text{ for } \theta < 10^0.$$

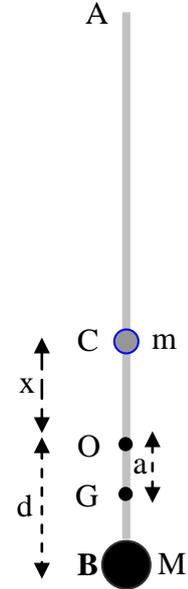


Fig.1

A- Theoretical study

1- Show that the position of G is given by: $a = \frac{Md - mx}{(M + m)}$.

2- Find the expression of the moment of inertia I of the pendulum about the axis (Δ) in terms of m, x, M and d.

3- The pendulum thus formed is deviated by an angle θ_0 from its equilibrium position and then released from rest at the instant $t_0 = 0$. The pendulum then oscillates around the stable equilibrium position. At an instant t, the position of the pendulum is defined by the angular abscissa θ , the angle that the vertical through O makes with OG, and its angular velocity is

$$\theta' = \frac{d\theta}{dt}.$$

a) Write, at the instant t, the expression of the kinetic energy of the pendulum in terms of I and θ' .

b) Show that the expression of the gravitational potential energy of the system (pendulum, Earth) is $P.E = - (M + m) g a \cos\theta$.

c) Write the expression of the mechanical energy of the system (pendulum, Earth) in terms of M, m, g, a, θ , I and θ' .

d) Derive the second order differential equation in θ that governs the motion of the pendulum.

e) Deduce that the expression of the proper period, for oscillations of small amplitude, has the

$$\text{form: } T = 2\pi \sqrt{\frac{I}{(M + m)ga}}.$$

f) Find the expression of the period T, in terms of M, m, d, g and x.

B- Application: metronome

A metronome is an instrument that allows adjusting the speed at which music is played.

The compound pendulum studied in part A represents a metronome where $M = 50 \text{ g}$, $m = 5 \text{ g}$, and $d = 2 \text{ cm}$. The graph of figure 2 represents the variations of the period T of this metronome as a function of the distance x.

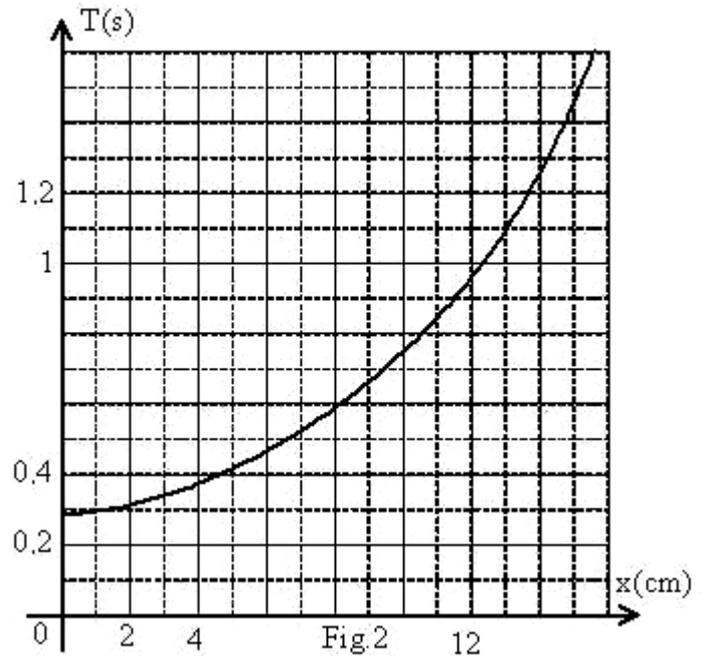
1) Find, in this case, the expression of the period T of the metronome as a function of x.

2) The leader of the orchestra (conductor), using a metronome to play a distribution, changes the position of C along OA, to follow the rhythm of the musical piece.

The rhythm is indicated by terms inherited from Italian for the classical distribution:

Name	Indication	Period (in s)
Grave	very slow	$T = 1.5$
Lento	Slow	$1 \leq T \leq 1.1$
Moderato	Moderate	$0.6 \leq T \leq 0.75$
Prestissimo	very fast	$0.28 \leq T \leq 0.42$

Determine, using a method of your choice, the positions between which the leader of the orchestra may move C to adjust the speed to the rhythm **Lento**.



Second exercise (7.5 points) Determination of the capacitance of a capacitor

In order to determine the capacitance C of a capacitor, we consider two experiments.

A- First experiment

We place the capacitor in series, with a coil of inductance $L = 0.32 \text{ H}$, a resistor of resistance $R = 100 \Omega$ and a low frequency generator G (LFG) that delivers across its terminals an alternating sinusoidal voltage: $u_g = u_{DB} = 8\sin(100\pi t - \pi/3)$ (u_g in V ; t in s) (Fig.1).

As a result, the circuit carries an alternating sinusoidal current of value: $i = I_m \sin(100\pi t)$ (i in A; t in s).

An oscilloscope is connected so as to display, on channel Y_1 , the voltage $u_{\text{coil}} = u_{AM}$ across the coil and, on channel Y_2 , the voltage $u_R = u_{MB}$ across the resistor.

The knob «Inv» (inverse) on channel Y_2 is pushed.

On the screen of the oscilloscope, we observe the waveforms (1) and (2) represented in figure 2.

The vertical sensitivity S_v is the same on the two channels: $S_v = 1 \text{ V/div}$. Take $0.32\pi = 1$.

1) Why did we push in the knob «Inv»?

2) Referring to figure 2:

- Determine the horizontal sensitivity S_h that is selected on the oscilloscope.
- Determine the phase difference between u_b and u_R .
- Which of the two voltages leads the other?
- Deduce that the coil has a negligible resistance.
- Determine the value of I_m .

3) Determine the expression of u_{coil} as a function of time t .

4) Show that expression of the voltage $u_C = u_{DA}$ across the

capacitor is given by $u_C = -\frac{I_m}{100\pi C} \cos 100\pi t$

5) Applying the law of addition of voltages, determine the value of C by giving t a particular value,

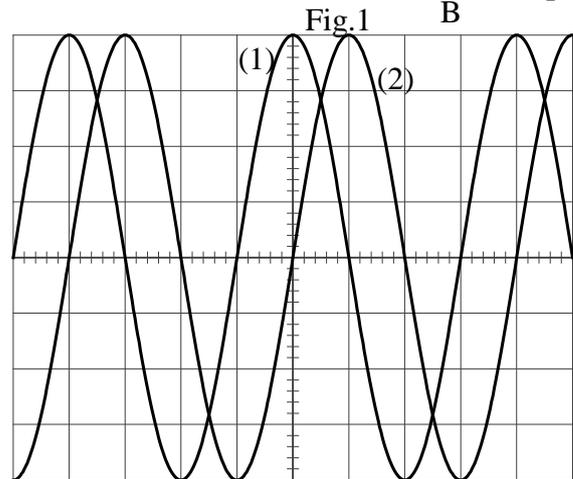
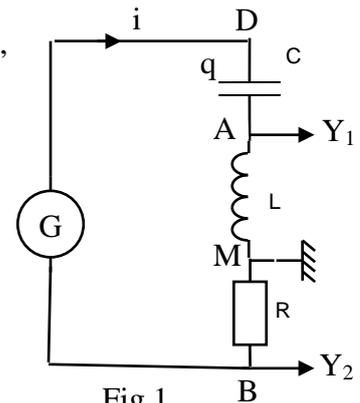


Fig.2

B- Second experiment

The capacitor, initially charged, is now connected across the coil of inductance $L = 0.32 \text{ H}$ (Fig.3). The oscilloscope, adjusted on the horizontal sensitivity $S_h = 2 \text{ ms/div}$, allows to display the voltage u_C across the capacitor (Fig.4).

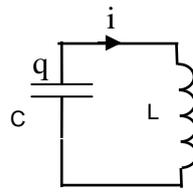


Fig.3

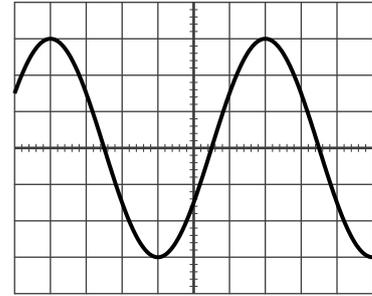


Fig.4

- 1) a) Show that the voltage u_C is sinusoidal of period T .
- b) Determine T in terms of L and C .
- 2) Calculate the value of C .

Third exercise (7.5 points) Index of refraction of a piece of glass

Consider a glass sheet of thickness $e = 5 \mu\text{m}$ and of index of refraction n , and a source S of white light having a filter so that Young's apparatus receives monochromatic light of wavelength λ in air of adjustable value. The object of this exercise is to study how the index n varies with λ .

A- Light interference – Interfringe distance.

Young's slits apparatus is formed of two very thin slits F_1 and F_2 , parallel and separated by a distance $a = 0.1 \text{ mm}$, and a screen of observation (E) placed parallel to the plane of the slits at a distance $D = 1 \text{ m}$ from this plane.

1) F_1 and F_2 are illuminated with a monochromatic radiation of wavelength λ issued from S that is placed at equal distances from F_1 and F_2 .

a) F_1 and F_2 must have two basic properties for the phenomenon of interference to be observed.

What are they?

b) Describe the system of fringes observed on (E).

c) At the point O of the screen, equidistant from F_1 and F_2 , we observe a bright fringe.

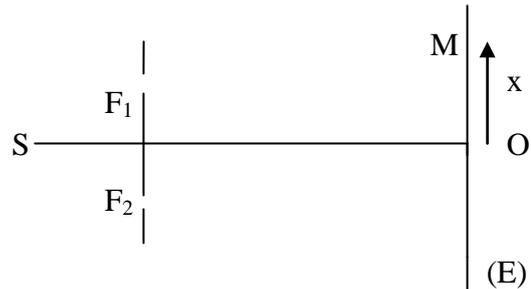
Why?

2) We admit that for a point M of (E), such that $OM = x$, the optical path difference in air or in

vacuum is given by $\delta = F_2M - F_1M = \frac{ax}{D}$.

a) Determine the expression of x_k corresponding to the center of the k^{th} bright fringe.

b) Deduce the expression of the interfringe distance i in terms of λ , D and a .



B- Introducing the sheet.

The glass sheet is put now just behind the slit F_1 . c and v are the speeds of light in vacuum (and practically in air) and in the glass sheet respectively.

1) Light crosses the glass sheet of thickness e during a time interval τ . Give the expression of τ in terms of e and v .

2) Give the expression of the distance d , covered by light in air during the time interval τ , in terms of n and e .

3) Deduce that the new optical path difference at point M is given by:

$$\delta' = F_2M - F_1M = \frac{ax}{D} - e(n - 1).$$

C- Measurement of n

N.B : Introducing the sheet does not affect the expression of the interfringe distance i

In this question the calculation of n must include 3 decimal places.

1) F_1 and F_2 are illuminated with a red radiation, of wavelength $\lambda_1 = 768 \text{ nm}$, issued from S .

The center of the central fringe is formed at O' , position that was occupied by the center of the 4^{th} bright fringe in the absence of the sheet. Determine the value of n_1 , the index of the sheet.

2) F_1 and F_2 are illuminated with a violet radiation of wavelength $\lambda_2 = 434 \text{ nm}$, issued from S .

The center of the central fringe is now formed at O'' , position that was occupied by the center of

the 8th dark fringe in the absence of the sheet. Determine the value of n_2 , the index of the sheet.

3) Can we consider the value of the index of refraction of a transparent medium without taking into account the radiation used? Why?

Fourth exercise (7.5 points) Nuclear Fission

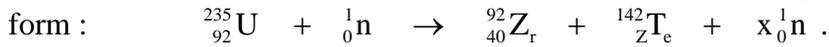
The object of this exercise is to show evidence of certain properties of nuclear fission.

Given: Masses of nuclei: $m(^{235}\text{U}) = 234.964 \text{ u}$; $m(^{92}\text{Zr}) = 91.872 \text{ u}$;

$$m(^{142}\text{Te}) = 141.869 \text{ u}; m(^1_0\text{n}) = 1.008\text{u}; 1\text{u} = 1.66 \times 10^{-27} \text{ Kg}; c = 3 \times 10^8 \text{ ms}^{-1};$$

A- Energy of fission.

One of the fission reactions of the uranium 235, in a nuclear power plant may be written in the form :



1) Determine Z and x specifying the laws used.

2) Calculate the energy produced by the fission of one nucleus of uranium 235.

3) Determine the mass of uranium 235 used in the power plant during one year, knowing that its useful electric power is 900 MW, and that its efficiency is 30 %.

B- Products of fission.

Among the products of fission, we find, in the core of the reactor, the radioelements: $^{137}_{55}\text{Cs}$ and $^{87}_{37}\text{Rb}$ of periods 30 years and 5×10^{11} years respectively.

These radioelements are placed in a pool called cooler. The nuclei $^{137}_{55}\text{Cs}$ and $^{87}_{37}\text{Rb}$ have the masses 137u and 87u respectively.

1) Suppose that 1 g of each of the radioelements is introduced into the pool at the instant $t_0 = 0$.

a) Calculate the number of nuclei of each of the radioelements at the instant $t_0 = 0$.

b) Deduce, for each radioelement, the number of nuclei remaining after 3 years stay in the pool.

c) Determine, for each radioelement, the number of decays per day at the moment of taking them out of the pool (3 years later).

2) Assuming that the danger of a radioelement on man depends on the radiations accumulated per day, which, of the two radioelements, is more dangerous? Justify.

C- Probability of fission.

In a physics dictionary, we read that the probability of a nucleus ^A_ZX to become fissionable is

proportional to the ratio $\frac{Z^2}{A}$, called the stability factor of a nucleus. This probability is no more

zero when this ratio exceeds 35.

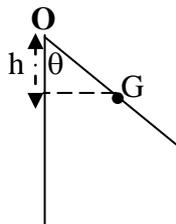
1) What do each of Z and A of the nuclide ^A_ZX represent?

2) Show that a nucleus must contain a number of neutrons N such that $N < \frac{Z(Z-35)}{35}$, so that

the probability to become fissionable is not zero.

3) Find the maximum number of nucleons that must be contained in a uranium nucleus, of $Z = 92$, so that the probability to undergo fission is not zero.

الدورة العادية للعام ٢٠٠٨	امتحانات الشهادة الثانوية العامة الفرع : علوم عامة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الفيزياء المدة ثلاث ساعات	مشروع معيار التصحيح

Part of the Q	Answer	Mark	
First exercise (7.5 points)			
A.1	$(M-m)\vec{OG} = M\vec{OB} + m\vec{OC} \Rightarrow a = \frac{Md - mx}{M + m}$.	0.75	
A.2	$I = I_M + I_m = Md^2 + mx^2$.	0.50	
A.3.a	$E_C = \frac{1}{2} I \dot{\theta}^2$.	0.50	
A.3.b	$E_{PP} = -(M+m)gh = -(M+m)gac \cos \theta$.		1.00
A.3.c	$E_m = E_C + E_{PP} = \frac{1}{2} I \dot{\theta}^2 - (M+m)gac \cos \theta$.	0.25	
A.3.d	$\frac{dE_m}{dt} = 0 = I \ddot{\theta} + (M+m)ga \dot{\theta} \sin \theta \Rightarrow \ddot{\theta} + \frac{(M+m)ga}{I} \sin \theta = 0$.	1.00	
A.3.e	for small θ , $\sin \theta = \theta \Rightarrow \ddot{\theta} + \frac{(M+m)ga}{I} \theta = 0$ \Rightarrow the proper angular frequency is: $\omega = \sqrt{\frac{(M+m)ga}{I}}$; the proper period $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi \sqrt{\frac{I}{(M+m)ga}}$.	1.25	
A.3.f	$T = 2\pi \sqrt{\frac{Md^2 + mx^2}{g(Md - mx)}}$.	0.75	
B.1	$T = \sqrt{\frac{0.08 + 20x^2}{1 - 5x}}$.	0.50	
B.2	graphically or by calculus: For $T = 1$ s, $x = 12.3$ cm. For $T = 1.1$ s, $x = 13$ cm. $\Rightarrow 12.3 < x(\text{cm}) < 13$.	1.00	
Second exercise (7.5 points)			
A.1	To eliminate the phase opposition obtained from the way with which the coil and the resistor are connected to the oscilloscope. (or: the oscilloscope, as it is connected, displays the voltage u_{BM} , but as we want to display u_{MB} , then we have to push the knob inversion.)	0.25	
A.2.a	The angular frequency of the voltage is $\omega = 100\pi$ rad/s; or the period is $T = \frac{2\pi}{\omega} = 0.02\text{s} = 20\text{ms}$; T covers 4 divisions on the screen $\Rightarrow S_h = \frac{20}{4} = 5\text{ms/div}$.	0.75	
A.2.b	T covers 4 divisions that correspond to an angle of 2π rad, the phase	0.75	

	difference φ is represented by 1 division $\Rightarrow \varphi = \frac{2\pi \times 1}{4} = \frac{\pi}{2}$ rad.	
A.2.c	u_b leads u_R .	0.25
A.2.d	Because the voltage across the coil of zero resistance leads by $\frac{\pi}{2}$ the current that flows through it.	0.50
A.2.e	$RI_m = 4 \text{ div} \times 1V / \text{div} = 4V \Rightarrow I_m = \frac{4}{100} = 0.04A$.	0.50
A.3	$u_b = L \frac{di}{dt} = 0.32 \times 0.04 \times 100\pi \cos(100\pi t) = 4\cos(100\pi t)$.	0.75
A.4	$i = \frac{dq}{dt} = C \frac{du_C}{dt} \Rightarrow u_C = \frac{1}{C} \times \text{primitive of } i = -\frac{I_m}{100\pi C} \cos(100\pi t)$ $\Rightarrow u_C = -\frac{1.28 \times 10^{-4}}{C} \cos(100\pi t)$.	0.75
A.5	$u_g = u_C + u_b + u_R \Rightarrow$ $8\sin(100\pi t - \frac{\pi}{3}) = -\frac{1.28 \times 10^{-4}}{C} \cos(100\pi t) + 4\cos(100\pi t) + 4\sin(100\pi t)$ For $t = 0$ we have : $-4\sqrt{3} = -\frac{1.28 \times 10^{-4}}{C} + 4 + 0 \Rightarrow C = 11.7 \times 10^{-6} \text{ F}$.	1.25
B.1.a	$u_C = u_b \Rightarrow u_C = L(\frac{di}{dt}) = -L\dot{q} = -LC\ddot{u}_C \Rightarrow \ddot{u}_C + \frac{1}{LC} u_C = 0 \Rightarrow$ the solution of such form of differential equation is sinusoidal of period T.	0.50
B.1.b	The angular frequency ω of motion is such that $\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$; the period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$.	0.50
B.2	From the waveform of figure 4 we have: $T = 6 \text{ div} \times 2\text{ms} / \text{div} = 12 \text{ ms} = 0.012\text{s}$. $C = \frac{T^2}{4\pi^2 L} = \frac{144 \times 10^{-6}}{12.5} = 11.5 \times 10^{-6} \text{ F}$.	0.75
Thierd exercise (7.5 points)		
A.1.a	F_1 and F_2 become synchronous and coherent	0.50
A.1.b	The interference fringes are bands that are equidistant, alternately dark and bright. These fringes are parallel to the slits	0.75
A.1.c	We have $\delta = 0$ then the radiations from F_1 and F_2 arrive at O in phase thus they form at O a bright fringe.	0.50
A.2.a	For bright fringes: $\delta = k\lambda \frac{ax}{D} = k\lambda \Rightarrow x_k = \frac{k\lambda D}{a}$;	0.50
A.2.b	$i = x_{k+1} - x_k = \frac{\lambda D}{a}$.	0.50
B.1	$\tau = \frac{e}{v}$	0.5
B.2	$d = c\tau = c \frac{e}{v} = ne$	0.50
B.3	the increase in the optical path for the light crossing the plate is: $ne - e = e(n-1)$ $\delta' = F_2 M - (F_1 M + e(n-1)) = \frac{ax}{D}$.	1.00

C.1	For the central fringe: $\delta' = 0 \Rightarrow \frac{ax_0}{D} - e(n_1 - 1) = 0$; so that $x_0 = 4i_1 = 4 \frac{\lambda_1 D}{a} \Rightarrow n_1 = 1 + \frac{4\lambda_1}{e} = 1.614$	1.25
C.2	$x_0 = 7.5 i_2 \Rightarrow n_2 = 1 + \frac{7.5\lambda_2}{e} = 1.651$	0.75
C.3	Non : $\lambda_1 \neq \lambda_2 \Rightarrow n_1 \neq n_2$ Yes : $\lambda_1 \neq \lambda_2 \Rightarrow n_1 \sqcup n_2$	0.75
Fourth exercise (7.5 points)		
A.1	${}^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{92}_{40}\text{Zr} + {}^{142}_{52}\text{Te} + x {}^1_0\text{n}$. Conservation of mass number: $235 + 1 = 92 + 142 + x \Rightarrow x = 2$ Conservation of charge number: $92 = 40 + Z \Rightarrow Z = 52$.	1.00
A.2	$\Delta m = 234.964 - 91.872 - 141.869 - 1.008 = 0.215 \text{ u}$ or $3.57 \times 10^{-28} \text{ kg}$. $E = \Delta m \cdot c^2 = 3.21 \times 10^{-11} \text{ J}$.	1.25
A.3	$E_1 = \frac{9 \times 10^8}{0.3} = 3 \times 10^9 \text{ J/s}$. The energy for one year : $3 \times 10^9 \times 365 \times 24 \times 3600 = 9.46 \times 10^{16} \text{ J}$. The number of nuclei undergoing fission: $\frac{9.46 \times 10^{16}}{3.21 \times 10^{-11}} = 2.947 \times 10^{27} \text{ nuclei}$. Its mass : $2.947 \times 10^{27} \times 234.964 \times 1.66 \times 10^{-27} = 1149.4 \text{ kg}$.	1.25
B.1.a	$N_0(\text{Cs}) = \frac{1}{137 \times 1,66 \times 10^{-24}} = 4,4 \times 10^{21} \text{ nuclei}$. $N_0(\text{Rb}) = \frac{1}{87 \times 1,66 \times 10^{-24}} = 6,9 \times 10^{21} \text{ nuclei}$	0.50
B.1.b	$N = N_0 e^{-\frac{0.693t}{T}}$. $\Rightarrow N(\text{Cs}) = 4.1 \times 10^{21} \text{ nuclei}$. $N(\text{Rb}) = 6.89 \times 10^{21} \text{ nuclei}$. $N(\text{Br}) = 0$	0.75
B.1.c	number of disintegrations per day = λN ($\lambda: \text{d}^{-1}$) For Cs : $\frac{0.693 \times 4.1 \times 10^{21}}{30 \times 365} = 2.6 \times 10^{17}$. For (Rb) : $\frac{0.693 \times 6.89 \times 10^{21}}{5 \times 10^{11} \times 365} = 2.6 \times 10^7$.	1.00
B.2	The more dangerous product is Cs, since its rate of disintegrations is greater.	0.25
c.1.a	Z is the charge number, A is the mass number.	0.25
C.2	$\frac{Z^2}{A} > 35 \Rightarrow \frac{Z^2}{Z + N} > 35$ $\Rightarrow Z(Z-35) > 35 N \Rightarrow N < \frac{Z(Z-35)}{35}$.	0.75
C.3	$\frac{Z^2}{A} > 35 \Rightarrow A < \frac{Z^2}{35} = \frac{(92)^2}{35} \approx 242$.	0.50