الدورة الإستثنائية للعام 2012	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل: أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I– (4 points)

In the table below, only one among the proposed answers to each question is correct.
Write down the number of each question and give, with justification, its corresponding answer.

N°	Questions	Answers		
		a	b	c
1	If $z=-\frac{1}{2}-\frac{i\sqrt{3}}{2}$ and $z'=-\frac{3}{2}+\frac{i\sqrt{3}}{2}$, then an argument of $(z-z')$ is	$\frac{\pi}{3}$	$\frac{5\pi}{3}$	$\frac{2\pi}{3}$
2	z is the affix of a point M. If $\lvert z-2i\rvert=\lvert z+4i\rvert$, then M moves on	a circle	a line parallel to the axis of ordinates	a line parallel to the axis of abscissas
3	One of the values of z verifying $\lvert z+1\rvert^2+\lvert z-1\rvert^2=2\lvert z+i\rvert^2$ is	3i	2 + 3i	2
4	The exponential form of $\frac{\cos\theta-i\sin\theta}{\sqrt{3}+i}$ is	$\frac{1}{2}e^{i\left(-\theta-\frac{\pi}{6}\right)}$	$2e^{i\left(-\theta-\frac{\pi}{6}\right)}$	$\frac{1}{2}e^{i\left(\theta-\frac{\pi}{6}\right)}$

II– (4 points)

The space is referred to a direct orthonormal system $(O\,;\vec{i},\vec{j},\vec{k})$.
Consider the points A (2; –2; –1) , B(l ; 0; –2), C(2; 1 ; –1), and the plane (P) with equation $x-2y+z+1=0$.

1) Show that $x-z-3=0$ is an equation of the plane (Q) determined by A, B and C.
2) a- Prove that (P) and (Q) are perpendicular and they intersect along the line (BC).
   b - Calculate the distance from A to (BC).
3) Let (d) be the line defined by:

$$\begin{cases} x=t-1 \\ y=t+1 \\ z=t+2 \end{cases} \quad \text{where t is a real parameter.}$$

   a- Verify that (d) is contained in (P).
   b- Let M be a variable point on (d). Prove that, as M moves on (d), the area of triangle MBC remains constant.

III– (4 points)

Consider two urns **U** and **V**.
Urn **U** contains eight balls: four balls numbered 1, three balls numbered 2 and one ball numbered 4.
Urn **V** contains eight balls: three balls numbered 1 and five balls numbered 2.

1) **Two balls are selected, simultaneously and randomly, from the urn U**.

   Consider the following events:
   - A : « the two selected balls have the same number »
   - B : « the product of the numbers on the two selected balls is equal to 4 ».

   Calculate the probability P(A) of the event A, and show that P(B) is equal to $\frac{1}{4}$.

2) **One of the two urns U and V is randomly chosen, and then two balls are simultaneously and randomly selected from this urn.**

   Consider the following events:
   - E : « the chosen urn is **V** »
   - F : « the product of numbers on the two selected balls is equal to 4 ».

   a- Verify that $P(F \cap E) = \frac{5}{28}$ and calculate $P(F \cap \overline{E})$.

   b- Deduce P(F).

3) **One ball is randomly selected from U, and two balls are randomly and simultaneously selected from V.**
   Calculate the probability of the event H: « the product of the three numbers on the three selected balls is equal to 8 ».

IV– (8 points)

Let f be the function defined, over] 1 ; + ∞ [, by $f(x) = \ln\left(\frac{x+1}{x-1}\right)$.

Denote by (C) its representative curve in an orthonormal system $\left(O; \vec{i}, \vec{j}\right)$.

1) Determine $\lim\limits_{x \to 1} f(x)$ and $\lim\limits_{x \to +\infty} f(x)$. Deduce the asymptotes to (C).

2) Verify that $f'(x) = \frac{-2}{(x-1)(x+1)}$ and set up the table of variations of f.

3) Draw (C).

4) a- Prove that f has an inverse function g whose domain of definition is to be determined.

   b- Prove that $g(x) = \frac{e^x+1}{e^x-1}$.

   c- (G) is the representative curve of g in the same as that of (C). Draw (G).

5) Let h be the function defined over] 1 ; + ∞ [by $h(x) = x\,f(x)$.

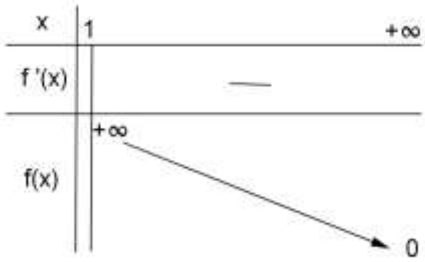
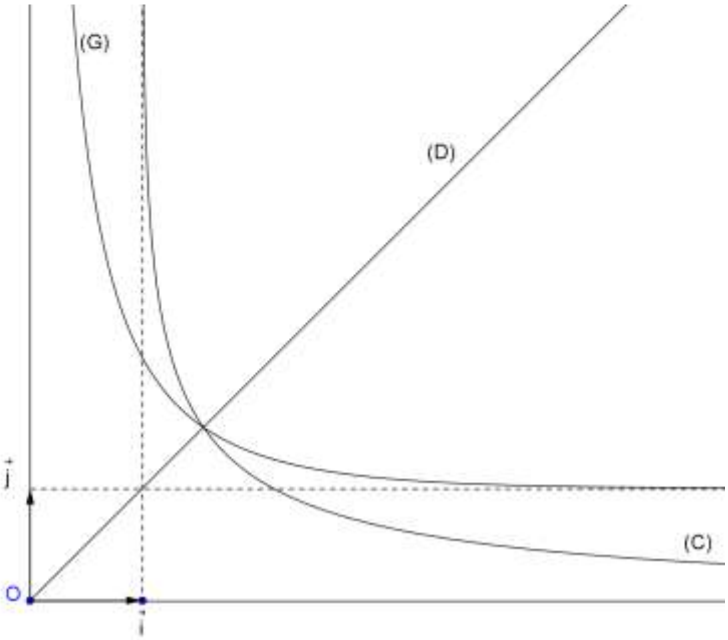
   a- Verify that $f(x) = h'(x) + \frac{2x}{x^2-1}$ and determine, over] 1 ; + ∞ [, an antiderivative F of f.

   b- Calculate the area of the region bounded by (C), the x-axis and the two lines with equations $x = 2$ and $x = 3$.

I	Solution		Mark
1	$z-z'=1-i\sqrt{3},\ z-z'=2e^{-i\frac{\pi}{3}}=2e^{i\frac{5\pi}{3}}$.	**b**	1
2	if A(2i) and B(– 4i) then $\lvert z-2i\rvert=\lvert z+4i\rvert \Leftrightarrow AM=BM,$ hence M moves on the perpendicular bisector of [AB] which is parallel to the axis of abscissas.	**c**	1
3	If z = 2 then 9 + 1 = 2(2^2 + 1) (true),	**c**	1
4	$\frac{\cos\theta-i\sin\theta}{\sqrt{3}+i}=\frac{e^{-i\theta}}{2e^{i\frac{\pi}{6}}}=\frac{1}{2}e^{i\left(-\theta-\frac{\pi}{6}\right)}$	**a**	1

II	Solution	Mark
1	$\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC})=0$ so an equation of (Q) is : x – z – 3 = 0 . **Or** : prove that the coordinates of A, B and C verify the given equation.	0.5
2a	$\vec{N}$ (1 ; –2 ; 1) is a normal vector to (P) ; $\vec{N'}$ (1 ; 0 ; –1) is a normal vector to (Q) and $\vec{N}\,.\,\vec{N'}=0$. (P) and (Q) are perpendicular. The coordinates of B and C verify the equation of (P).	1
2b	$d_{A/(BC)}=d_{A/(P)}=\frac{\lvert 2+4-1+1\rvert}{\sqrt{1+4+1}}=\frac{6}{\sqrt{6}}=\sqrt{6}$. **Or** : $d=\frac{\lVert\overrightarrow{AB}\wedge\overrightarrow{BC}\rVert}{\lVert\overrightarrow{BC}\rVert}=\sqrt{6}$.	1
3a	t – 1 – 2t – 2 + t + 2 + 1 = 0 so (d) is in (P).	0.5
3b	$\overrightarrow{BC}$ (1 ; 1 ; 1) is a direction vector to(d) then (d) // (BC) and the distance from M to (BC) is constant, so the area of triangle MBC remains constant. **Or** : calculate the distance from M(t–1 ; t+1 ; t+2) to (BC) which is equal to the distance from M to (Q) and show that it is independent of t. **Or** : calculate the area of triangle MBC : $\frac{1}{2}\lVert\overrightarrow{MB}\wedge\overrightarrow{BC}\rVert=\frac{1}{2}\sqrt{54}$ = constant.	1

III	Solution	Mark
1	P(A) $=\frac{C_4^2}{C_8^2}+\frac{C_3^2}{C_8^2}=\frac{9}{28}$; P(B) $=\frac{C_3^2}{C_8^2}+\frac{C_4^1\times C_1^1}{C_8^2}=\frac{7}{28}=\frac{1}{4}$	1
2a	$P(F\cap E)=P(E)\times P(F/E)=\frac{1}{2}\times\frac{C_5^2}{C_8^2}=\frac{5}{28}$. $P(F\cap\overline{E})=P(\overline{E})\times P(F/\overline{E})=\frac{1}{2}\times P(B)=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$.	1.5
2b	P(F) = $P(F\cap E)+P(F\cap\overline{E})=\frac{5}{28}+\frac{1}{8}=\frac{17}{56}$.	0.5
3	P(product = 8) = P(2 ;{2 ,2}) + P(4 ;{2 ,1})= $\frac{3}{8}\times\frac{C_5^2}{C_8^2}+\frac{1}{8}\times\frac{C_3^1\times C_5^2}{C_8^2}=\frac{45}{224}$.	1

IV	Solution	Mark
1	$\lim_{x\to1} f(x) = \lim_{x\to1} \ln\left(\frac{x+1}{x-1}\right) = +\infty$ and $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \ln\left(\frac{x+1}{x-1}\right) = 0$ The lines with equations x = 1 and y = 0 are the asymptotes to (C).	1.5
2	$f'(x) = \frac{u'}{u} = \frac{\frac{-2}{(x-1)^2}}{\frac{(x+1)}{x-1}} = \frac{-2}{(x-1)(x+1)} < 0$ 	1
3		1.5
4a	Over]1 ; +∞[; f is continuous and strictly decreasing, so it has an inverse function g defined over]0 ;+∞[.	0.5
4b	$f(g(x)) = x$ gives $\ln\frac{g(x)+1}{g(x)-1} = x$; $\frac{g(x)+1}{g(x)-1} = e^x$ so $g(x) = \frac{e^x+1}{e^x-1}$. **Or :** $g(f(x)) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \frac{2x}{2} = x$	1
4c	(G) is the symmetric of (C) with respect to line (D) with equation y = x. (See figure – question 3)	1
5a	$h'(x) = f(x) + xf'(x) = f(x) - \frac{2x}{(x-1)(x+1)}$ so $f(x) = h'(x) + \frac{2x}{x^2-1}$. $F(x) = h(x) + \ln(x^2-1) = x\ln\left(\frac{x+1}{x-1}\right) + \ln(x^2-1)$.	1.5
5b	$A = F(3) - F(2) = 3\ln2 + \ln8 - 2\ln3 - \ln3 = 2\ln8 - 3\ln3$; A = (2ln8 – 3ln3)u^2.	0.5