مسابقة في مادة الرياضيات الاسم:
المدة ساعتان الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (2 points)

Given :

$A = 2\sqrt{27}+3\sqrt{75}-3\sqrt{48}$ and $B = \dfrac{22}{\sqrt{18}-\sqrt{8}}$.

1) Write A in the form $a\sqrt{3}$ and B in the form $b\sqrt{2}$ where a and b are two integers.
2) Compare A and B and justify.
3) Show that $A - B = \dfrac{1}{A+B}$.

II- (2 points)

The questions 1) and 2) of this exercise are independent.

1) A class contains 30 students where 40 % of them are boys. Another class contains 20 students where 60 % of them are boys.
   The students of these two classes meet together in the computer room.
   Calculate the number and the percentage of boys in this room.
2) All the articles of a certain shop are subject to an increase of 20 % on their prices.
   Denote by x the original price of an article and by y its new price after the increase.
   a. Find y as a function of x.
   b. If the new price of a calculator is 30 000 LL, what is its original price?

III- (3 points)

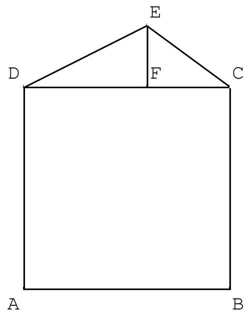
A first bunch of flowers is formed by 3 roses and 4 tulips and it costs 4800 LL.
A second bunch of flowers is formed by 5 roses and 6 tulips and it costs 7500 LL.
Denote by x the price of one rose, and by y the price of one tulip.

1) Write a system of two equations modeling the previous information.
2) Solve the previous system in showing the steps of calculation. Determine the price of one rose and that of one tulip.
3) A client buys a bunch formed by 10 flowers and he pays 6450 LL.
   Calculate the number of roses and that of tulips in this bunch .

IV-(3 points)

Consider the polynomial $P(x) = (x+9)^2 - 3(x-1)(x+9)$.

1) Factorize P(x).
2) In this part,the unit of length is the centimeter.
   In the figure to the right, ABCD is a square, DEC is a triangle such that CF = 9, DF = x and the height EF = x – 1 with x>1.
   Calculate x such that the area of the square ABCD is 6 times the area of the triangle CED.



V- (5 points)

Consider in an orthonormal system of axes x'ox and y'oy the line (d) with equation y = 3x + 2 and the two points A(1 ;5) and B(– 2 ;–4).

1) Show that the points A and B belong to the line (d).
2) Locate A and B and plot (d).
3) Let (d') be the perpendicular bisector of [AB] and H the midpoint of [AB].
   a. Calculate the coordinates of H.
   b. Determine the equation of (d').
4) Let M (– 5; 2) be a point on (d').
   a. Given that MA $=3\sqrt{5}$, justify that MB $=3\sqrt{5}$.
   b.Calculate AB and deduce that AMB is a right isosceles triangle.
5) Consider the point P on the line (d') so that P is distinct from M and AP = AM.
   a. Locate P, and show that BP = BM.
   b. What is the nature of the quadrilateral MAPB ? Justify.

VI- (5 points)

Consider a circle (C) with center O, diameter [AB] and radius 2 cm. T is a point on (C) so that AT = 2 cm , and M is the symmetric of O with respect to A.

1) a. Make a figure.
   b. Prove that (MT) is tangent to (C).
   c. Calculate MT.
   d.Prove that MTB is an isosceles triangle.
2) E is the meeting point of (MT) and the tangent at B to (C).
   a. Prove that T is the midpoint of [EM].
   b. (TO) intersects (C) at F, calculate EF.
   c. Calculate to the nearest degree the angle $\widehat{EFT}$.
3) N is a variable point on (C) and S is the image of N under the translation with vector $\overrightarrow{AM}$.
   a. Prove that ASNO is a rhombus.
   b. K is the midpoint of [MS], prove that K moves on a fixed circle whose diameter is to be determined.

I- (2 points)

Part of Q.	correction	Mark
1	$A=9\sqrt{3}$, $B=11\sqrt{2}$	1
2	A>B	0.5
3	$A^2=243; B^2=242; A^2-B^2=1$	0.5

II- (2 points)

1	Number of boys of the room: $\frac{30\times40}{100}+\frac{20\times60}{100}$ = 24. Percent of boys is: $\frac{24}{50}\times100$ = 48that is 48 %.	**1**
2.a	y = 1,2 x.	**0.50**
2.b	The original price is: $\frac{30000}{1,2}$ = 25000 LL.	**0.50**

III- (3 points)

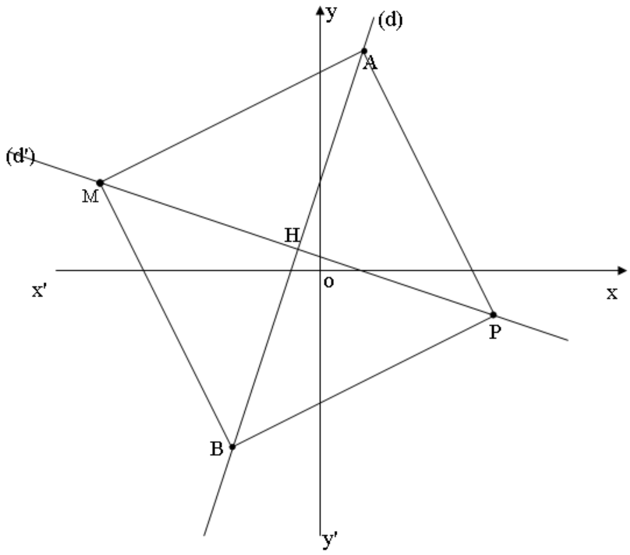
1	$\begin{cases}3x+4y=4800\\5x+6y=7500\end{cases}$	**0.75**
2	x = 600 et y = 750 The price of one rose is 600 LL and that of one tulip is 750 LL.	**1**
3	the number of roses is 7 and that of tulips is 3..	**1.25**

IV- (3 points)

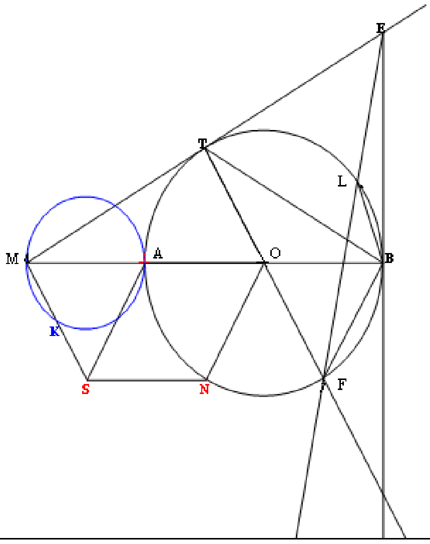
1	P(x) = (x + 9) (– 2x + 12) = -2(x+9)(x-6)	**1**
2	The area of the square = $(x + 9)^2$; 6 time the area of the triangle CED = 3(x – 1) (x + 9). P(x) = 0 x = – 9 uinacceptable x = 6 acceptable	**2**

V- (5 points)

1	A belongs to (d).; B belongs to (d)	0.50
2	(d) (AB)	0.50
3.a	H $\left(-\frac{1}{2};\frac{1}{2}\right)$.	0.50
3.b	a'= $-\frac{1}{3}$ and (d') passes through H therefore ; y = $-\frac{1}{3}$ x + $\frac{1}{3}$.	1
4.a	MA=$3\sqrt{5}$ = MB	0.50
4.b	AB^2 = 90 then AB=$\sqrt{90}=3\sqrt{10}$, MAB,MAB is a right isosceles triangle.	0.75
5.a	MA = MB = $3\sqrt{5}$ = AP and P belongs to (d') therefore PA = PB then PB = BM.	0.75
5.b	MAPB is a square.	0.50

VI- (5 points)

1.a		0.50
1.b	In the triangle MTO, [TA] is a median and TA = 2 = $\frac{1}{2}$ MO. Hence, MTO is right at T and (MT) is tangent to (C).	0.75
1.c	$MT^2 = MO^2 - OT^2 = 16 - 4 = 12$, then $MT = 2\sqrt{3}$.	0.50
1.d	$TB = MT = 2\sqrt{3}$, so the triangle MTB is isosceles with vertex T.	0.50
2.a	TAO is an equilateral triangle. $\widehat{ETB} = \widehat{EBT} = \widehat{TAB} = 60^o$. Hence, the triangle ETB is equilateral. So, ET = BT, but BT = TM, then TE = TM. Therefore T is the midpoint of [EM].	0.75
2.b	$EF^2 = ET^2 + TF^2 = 12 + 16 = 28$; thus, $EF = 2\sqrt{7}$.	0.50
2.c	$\widehat{TFE} \approx 41^o$ or 40^0.	0.50
3.a	ASNO is a parallelogram with OA = ON, then it is a rhombus.	0.50
3.b	MAS is an isosceles triangle and [AK] is a median, then $\widehat{AKM} = 90^o$. Hence, K moves on the circle with diameter [AM].	0.50