الدورة الإستثنائية للعام 2012

عدد المسائل: ستة مسابقة في مادة الرياضيات الاسم: المدة ساعتان الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات

-يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة

### I- (2 points)

Consider the following numbers A, B and C:

$$A = \left(\frac{2}{3}\right)^2 + \frac{1}{3}$$
;  $B = \frac{5}{3} - \frac{2}{3} \div \left(1 + \frac{3}{2}\right)$ ;  $C = \frac{18 \times 10^8}{8 \times 10^7 \times 3.5}$ 

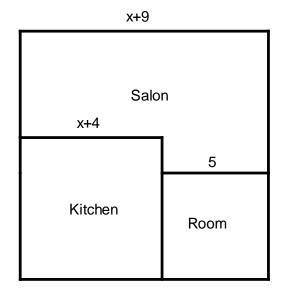
- 1) Write, showing all the steps of calculation, each of the numbers A, B and C as a fraction in its simplest form.
- 2) Out of the found fractions, indicate that which is decimal. Justify.

# II- (3 points)

Consider the following expressions:

$$E=(x+9)^2-25$$
;  $G=(x+4)(x+14)-2(x+4)^2$ .

- 1) Verify that E=(x+4)(x+14) and factorize G.
- The adjoining diagram is that of an apartment in the form of a square with side (x+9) meters (x ≥ 0).
  It is formed of a salon, a room and a kitchen.
  The room is a square with side 5 meters and the kitchen is also a square with side (x+4) meters.
  - **a.** Express ,in terms of x, the area A of the apartment and calculate the area of  $A_1$  of the room.
  - **b.** Determine  $A_2$ , the sum of areas of the salon and the kitchen
  - **c.** Express ,in terms of x, the area  $A_3$  of the kitchen. Determine x so that  $A_2$  is the double of  $A_3$ .



## III- (3 points)

The following three questions are independent:

- Solve the following equation and give the answer in the form  $a\sqrt{b}$  where a and b are two integers:  $\sqrt{2}(x-1) = 2(x-2) + 3\sqrt{2}$ .
- 2) The measures, in cm, of sides of a triangle ABC are :  $AB = \sqrt{7} + 1$ ,  $BC = \sqrt{7} 1$  and AC = 4. Show that ABC is a right triangle.
- 3) An article costs 18 000LL. If its price is subject to a discount of 12%, followed by a raise of 15%, what is therefore the new price of this article?

#### IV- (2 points)

The owner of a bookshop proposes the following offer to his clients:

"The first five CD are rented at the rate of 600 LL each, and the others are rented at the rate of 500 LL each".

A client has rented x CD and paid a sum less than 9 000 LL.(x>5).

- 1) Show that the previous information are modeled by the following inequality: 500x + 500 < 9000.
- 2) Solve this inequality and find the greatest value of x.

#### V- (5 points)

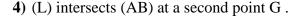
In an orthonormal system of axes x'Ox, y'Oy, consider the points A(-1;0); B(0;2) and E(3;-2).

- 1) Plot A, B, and E in this system.
- 2) a. Prove that BE = 5.
  - **b.** Let I be the midpoint of [BE]. Calculate the coordinates of I.
  - **c.** Calculate AI and deduce that the triangle ABE is right at A.
- 3) Denote by (C) the circle circumscribed about triangle ABE and by (t) the tangent at B to (C).
  - **a.** Verify that the slope of (BE) is equal to  $-\frac{4}{3}$ .
  - **b.** Write the equation of (t).
  - **c.** (t) intersects x'Ox at F. calculate, rounded to the nearest degree, the measure of the angle BFI.

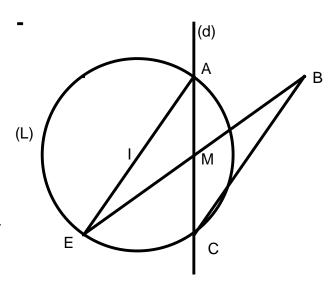
## VI- (5 points)

In the adjoining figure:

- A and B are two fixed points
- (d) is the perpendicular at A to (AB)
- C is a variable point on (d)
- M is the midpoint of [AC]
- E is the symmetric of B with respect to M
- (L) is the circle with diameter [AE] and center I.
- 1) Reproduce this figure.
- 2) Prove that the quadrilateral ABCE is a parallelogram.
- 3) Let F be the translate of B under the translation with vector  $\overrightarrow{CA}$ . Show that E, A and F are collinear.



- **a.** Prove that ACEG is a rectangle. Deduce that G is the translate of A under the translation with vector  $\overrightarrow{BA}$ .
- **b.** Prove that the two triangles AGM et BGF are similar .
- 5) What is the locus of point I as C moves on (d)?



مسابقة في مادة الرياضيات

مشروع معيار التصحيح

	Part of Q.	correction	Note
I	1	$A = \frac{7}{9}$ ; $B = \frac{5}{3} - \frac{4}{15} = \frac{7}{5}$ ; $C = \frac{45}{7}$ .	1.5
	2	B is decimal, (denominator is 5).	0.5
II	1	E = (x+9+5)(x+9-5) = (x+4)(x+14) $G = (x+4)[x+14-2(x+8)] = (x+4)(-x+6)$	1
	2.a	$A=(x+9)^2$ $A_1=25$	0.5
	2.b	$A_2 = (x+9)^2 - 25$	0.5
	2.c	$A_3$ = $(x+4)^2$ . $A_2$ - $2A_3$ =0 , G=0, x= -4 not acceptable $x$ = 6 acceptable	1
III	1	$x(\sqrt{2}-2)=4\sqrt{2}-4$ ; $x=\frac{4(\sqrt{2}-1)}{\sqrt{2}-2}$ then $x=-2\sqrt{2}$	1
	2	$(\sqrt{7} + 1)^2 = 8 + 2\sqrt{7}$ ; $(\sqrt{7} - 1)^2 = 8 - 2\sqrt{7}$ AB <sup>2</sup> + BC <sup>2</sup> = AC <sup>2</sup> , then ABC is right at B.	1
	3	The discount price is 15 840LL. Then its new price after raise is 18 216 LL.	1
IV	1	$5 \times 600 + (x - 5) \times 500 < 9000$ ; so $,500x + 500 < 9000$ .	1.25
	2	$500x < 8\ 500\ ; \qquad \text{then}\ x < 17.$ The greatest value of x is 16	0.75

V		A,B and E	0.5
	2.a	BE = 5.	0.75
	2.b	$I(\frac{3}{2};0),$	0.5
	2.c	$AI = \frac{5}{2} = \frac{BE}{2}$ , BAE right at A.	1
	3.a	Slope of (BE) = $-\frac{4}{3}$	0.5
	3,b	Slope of (t) = $\frac{3}{4}$ ,(t) passes through B(0;2) its equation is $y = \frac{3}{4}x + 2$	1
	3.c	$\tan^{-1}\left(\frac{3}{4}\right) \approx 36^{\circ}, 8$ so BFI $\approx 37^{\circ}$	0.75
VI	1	(L) (d) B	0.25
	2	[BE]and [CA] bisect each other , ABCE parallelogram.	0.75
	3	E, A and F collinear bec	1
	4.a	ACEG is a rectangle bec: $AG = BA$ bec	1
	4.b	ACEG is a rectangle bec: AG = BA bec $\frac{AM}{BF} = \frac{AG}{BG} = \frac{1}{2}; \qquad GAM = GBF = 90^{\circ}$	1
	5	I moves on then perpendicular bisector of [AG] bec	1