مسابقة في مادة الفيزياء
المدة : ثلاث ساعات

الاسم:
الرقم:

This exam is formed of four exercises in four pages.
The use of non-programmable calculator is recommended.

First exercise: (7.5 points)

Charging and Discharging of a Capacitor

The aim of this exercise is to study the charging and the discharging of a capacitor of capacitance C =1 μF. For that we connect the circuit of figure 1 which is formed of the capacitor, an ideal generator of constant voltage E, a resistor of resistance R and a double switch (K).

Take the direction of the current as a positive direction.

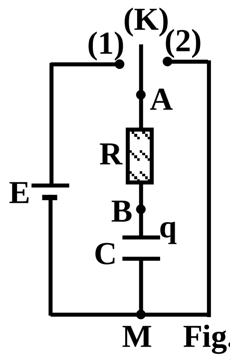


Fig.1

A – Charging of the capacitor

The capacitor is initially neutral and the switch (K) is turned to position (1) at the instant $t_0 = 0$.

A convenient apparatus records the variation of the voltage $u_C = u_{BM}$ across the terminals of the capacitor as a function of time.

1) Derive the differential equation that describes the variation of the voltage u_C as a function of time.

2) The solution of the differential equation is given by:

$u_C = A + B\,e^{-\frac{t}{\tau}}$, where A, B and τ are constants.
Determine the expressions of these constants in terms of R, C and E.

3) Figure 2 shows the variation of u_C as a function of time t. The straight line OT represents the tangent to the curve $u_C(t)$ at $t_0 = 0$.

a) Determine the value of τ.

b) Deduce the values of E and R.

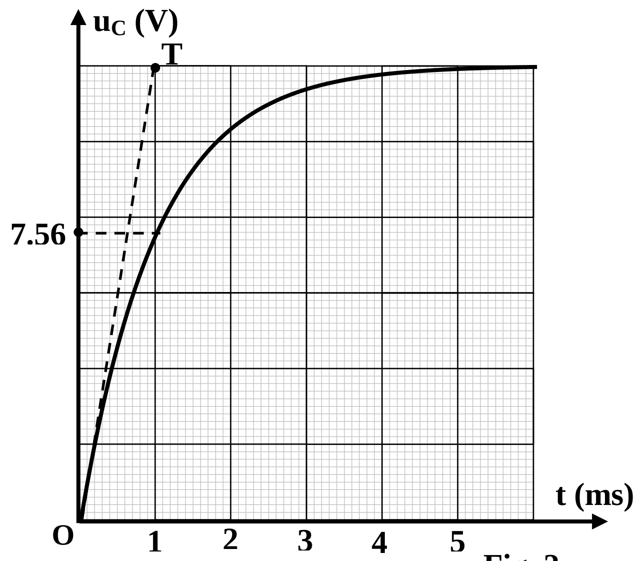


Fig. 2

B – Discharging of the capacitor

The charging of the capacitor being completed, the switch (K) is turned to position (2) at a new origin of time $t_0 = 0$.

At an instant t the circuit carries a current i.

1) Redraw the figure of the discharging circuit and indicate on it the direction of the current i.

2) Show that the differential equation in i has the form:

$i + RC\frac{di}{dt} = 0$.

3) Verify that $i = I_0\,e^{-\frac{t}{\tau}}$ is a solution of this differential equation, where $I_0 = \frac{E}{R}$.

4) **a)** Calculate the value of i at $t_0 = 0$ and at $t_1 = 2.5\,\tau$.
b) Deduce the value of u_C at $t_1 = 2.5\,\tau$.

5) Determine the electric energy W_e lost by the capacitor between $t_0 = 0$ and $t_1 = 2.5\,\tau$.

6) The energy dissipated due to joule's effect in the resistor between t_0 and t_1, is given by $W_h = \int_{t_0}^{t_1} R\,i^2\,dt$.

a) Determine the value of W_h.

b) Compare W_h and W_e. Conclude.

<u>Second exercise</u>: (7.5 points)

Determination of the inductance of a coil and the capacitance of a capacitor

The aim of this exercise is to determine the inductance L of a coil of negligible resistance and the capacitance C of a capacitor.
For this aim we perform two experiments:

A – First experiment

In this experiment, we set up the circuit represented in figure 1. This series circuit is composed of: a resistor (D_1) of resistance $R_1 = 25\ \Omega$, the coil of inductance L and of negligible resistance and an (LFG) maintaining across its terminals an alternating sinusoidal voltage of expression:
$u_{AB} = U_m \sin \omega t$ (u_{AB} in V , t in s).

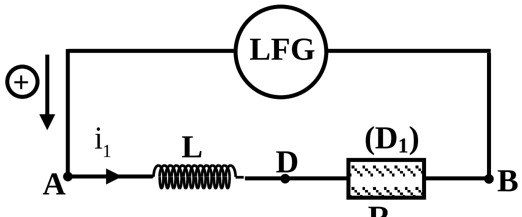


Fig.1

The circuit thus carries an alternating sinusoidal current i_1 .
An oscilloscope is used to display the variation, as a function of time, of the voltage u_{AB} on channel (Y_1) and the voltage u_{DB} on channel (Y_2).
The adjustments of the oscilloscope are:

- vertical sensitivity for the both channels: 1 V/div;
- horizontal sensitivity: 1 ms/div.

1) Redraw figure (1) and show on it the connections of the oscilloscope.
2) The obtained waveforms are represented on figure (2).
  a) The waveform (a) represents u_{AB} . Justify.
  b) Using the waveforms of figure (2), determine:
    i) the angular frequency ω of the voltage u_{AB} ;
    ii) the maximum value U_m and U_{m1} of the voltages u_{AB} and u_{DB} respectively;
    iii) the phase difference between u_{AB} and u_{DB} .

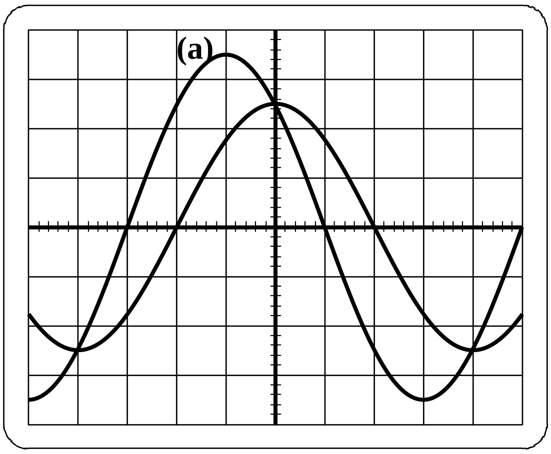


Fig. 2

3) **a)** Write the expression of the voltage u_{DB} as a function of time.
  b) Deduce that $i_1 = 0.1 \sin (\omega t - \frac{\pi}{4})$ (i_1 in A, t in s).
4) Determine the value of L by applying the law of addition of voltages.

B – Second experiment

In this experiment, another series circuit composed of: the capacitor of capacitance C, a resistor (D_2) and an ammeter (A_1) of negligible resistance, is connected between A and B as shown in figure 3. Thus the second branch carries an alternating sinusoidal current i_2 .
The oscilloscope is used, in this case, to display the voltage $u_{EB} = u_C$ across the terminals of the capacitor and the voltage u_{DB} across the terminals of (D_1).
U_m and ω of the (LFG) are kept constant. The adjustments of the oscilloscope remain the same.

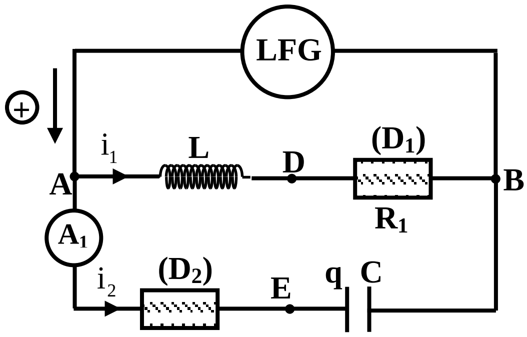


Fig.3

The obtained two waveforms are confounded and represented on figure 4.

Knowing that $i_1 = 0.1 \sin(\omega t - \frac{\pi}{4})$ (i_1 in A, t in s).

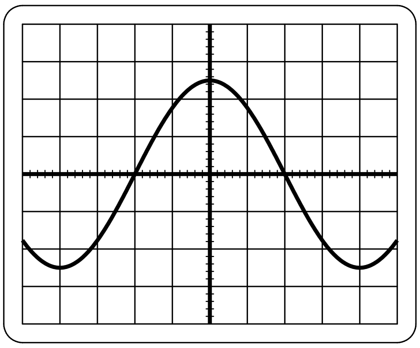


Fig. 4

1) Write the expression of u_C as a function of time.
2) Determine the expression of i_2 in terms of C and t.
3) The ammeter (A_1) indicates 27.7 mA. Determine the value of C.

<u>Third exercise</u>: (7.5 points)

Torsion Pendulum

The aim of this exercise is to study the motion of a torsion pendulum. Consider a torsion pendulum that is constituted of a homogeneous disk (D), of negligible thickness, suspended from its center of inertia O by a vertical torsion wire connected at its upper extremity to a fixed point O' (Fig.1).

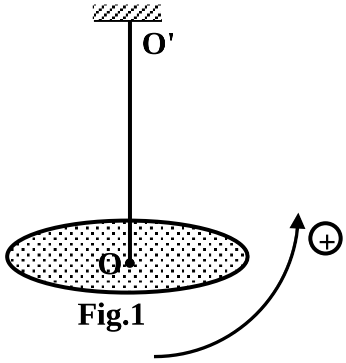


Fig.1

Given:

- the moment of inertia of (D) about the axis (OO'): $I = 3.2 \times 10^{-6}$ kg.m^2;
- the torsion constant of the wire: $C = 8 \times 10^{-4}$ m.N/rad;
- the horizontal plane passing through O is taken as a gravitational potential energy reference.

A – Free un-damped oscillations

The forces of friction are supposed negligible.
The disk is in its equilibrium position. It is rotated around (OO'), in the positive direction , by an angle $\theta_m = 0.1$ rad, the disk is then released without initial velocity at the instant $t_0 = 0$.

At the instant t, the angular abscissa of the disk is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

1) Write, at the instant t, the expression of the mechanical energy of the system (pendulum, Earth) in terms of I, C, θ and θ'.
2) Derive the second order differential equation that describes the variation of θ as a function of time.
3) The solution of this differential equation is of the form: $\theta = \theta_m \cos(\frac{2\pi}{T_0} t + \varphi)$.
Determine the constants T_0 and φ.

B – Free damped oscillations

In reality, the forces of friction are no more negligible. (D) thus performs slightly damped oscillations of pseudo period T.

1) At the end of each oscillation, the amplitude of the oscillations decreases by 2.5% of its precedent value.
  a) Calculate the mechanical energy E_0 of the system (pendulum, Earth) at the instant $t_0 = 0$.
  b) Show that the loss in the mechanical energy of the system (pendulum, Earth) by the end of the first oscillation is: $|\Delta E| = 1.97 \times 10^{-7}$ J.
2) Calculate the value of the average power dissipated by the resistive forces admitting that the value of the pseudo period T is equal to that of T_0.

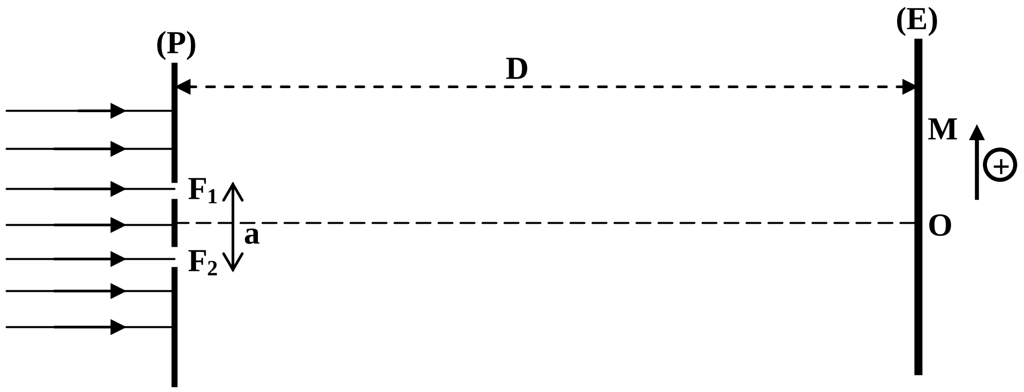
C – Driven oscillations

A driving apparatus (M) allows compensating for the loss of energy at the end of each oscillation. This apparatus stores energy W= 0.8 J. The energy furnished by (M) to drive the oscillations represents 25% of energy stored in it.
Determine, in days, the maximum duration of driving the oscillations.

Fourth exercise: (7.5 points)

Diffraction and interference

Two horizontal slits F_1 and F_2, are illuminated normally with a laser source. Each slit, cut in an opaque screen (P), has a width a_1 = 0.1 mm and are situated at a distance F_1F_2 = a = 1 mm from each other. The wavelength of the laser light is λ = 600 nm.
The distance between the plane (P) of the slits and the screen of observation (E) is D = 2 m. (Figure below).
O is a point on the screen (E) and belongs to the perpendicular bisector of $[F_1F_2]$.



A – We cover the slit F_1 by an opaque sheet thus light is emitted only from F_2.

1) The phenomenon of diffraction is observed on the screen (E). Justify.
2) Redraw the figure and trace the beam of light leaving the slit F_2.
3) Describe the pattern observed on the screen (E).
4) Write the expression of the angular width α (α is very small) of the central bright fringe in terms of λ and a_1.
5) **a)** Show that the linear width L of the central bright fringe is given by: $L = \frac{2\lambda D}{a_1}$.
b) Calculate L.
6) The opaque sheet is moved to cover the slit F_2. The slit F_1 sends light now on the screen (E). The center of the new central bright fringe is at a distance d from the previous center of the central bright fringe. Specify the value of d.

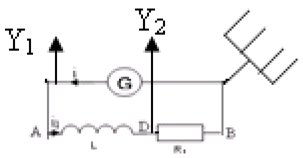
B – We remove away the opaque sheet and the two slits are now both illuminated with the laser beam.
For a point M on (E), such that $x = \overline{OM}$, the optical path difference in air is given by $\delta = \frac{ax}{D}$.

1) Determine the expression of the abscissa x_k corresponding to the center of the k^{th} dark fringe.
2) Deduce the expression of the interfringe distance i.
3) Calculate i.
4) Consider a point N on the screen (E) having an abscissa $x_N = \overline{ON}$ = 2.4 mm. Specify the nature and the order of the fringe at point N.
5) We move the screen (E) towards the plane (P) of the slits and parallel to it by a distance of 40 cm. Determine the nature and the order of the new fringe at N.

First exercise (7.5 points)

Part of the Q	Answer	Mark
A.1.	$E = u_R + u_C = Ri + u_C$; But $i = \frac{dq}{dt} = C\frac{du_C}{dt}$. $\Rightarrow RC\frac{du_C}{dt} + u_C = E$	0.75
A.2.	$u_C = A + Be^{-\frac{t}{\tau}} \Rightarrow \frac{du_C}{dt} = -\frac{B}{\tau}e^{-\frac{t}{\tau}}$ $E = -\frac{RCB}{\tau}e^{-\frac{t}{\tau}} + A + Be^{-\frac{t}{\tau}} \Rightarrow E = A + (1-\frac{RC}{\tau})Be^{-\frac{t}{\tau}}$ $\Rightarrow E = A, (1-\frac{RC}{\tau})Be^{-\frac{t}{\tau}}=0$ but $B \neq 0 \Rightarrow \tau = RC$ At t=0 ,$u_C = 0 \Rightarrow A + B = 0, \Rightarrow B = -A = -E$ $\Rightarrow u_C = E(1-e^{-\frac{t}{\tau}})$	1
A.3.a	From the graph , τ is the point where line OT intersects the asymptote $\Rightarrow \tau$= 1ms	0.5
A.3.b	At $t = \tau, u_C = 0.63E \Rightarrow E = \frac{7.56}{0.63} = 12V$ $\tau = RC \Rightarrow R = 10^3\Omega$	0.75
B.1.	Figure	0.25
B.2.	$u_{AB} + u_{BM} = 0, \Rightarrow -Ri + u_c = 0 \Rightarrow -Ri + \frac{q}{c} = 0$ Derive w.r.t.time , $-R\frac{di}{dt} + \frac{1}{C}\left(\frac{dq}{dt}\right)$ but $i = -\frac{dq}{dt} \Rightarrow i + RC\frac{di}{dt} = 0$	0.75
B.3.	$i = I_0e^{-\frac{t}{\tau}} \Rightarrow \frac{di}{dt} = -\frac{I_0}{\tau}e^{-\frac{t}{\tau}} \Rightarrow I_0e^{-\frac{t}{\tau}} - \frac{RC}{\tau}I_0e^{-\frac{t}{\tau}} = 0$, verified	0.5
B.4.a.	$i = I_0e^{-\frac{t}{\tau}}$, at $t_0 = 0, i = I_0e^0 = 0.012A \Rightarrow$ At t =2.5τ, $i = I_0e^{-\frac{t}{\tau}} = 0.082I_0 \Rightarrow i = 9.84\times10^{-4}A$	0.75
B.4.b	$u_C = u_R = Ri = 0.984V$	0.25
B.5.	$W_e = \frac{1}{2}C(E^2 - u^2) = 7.15\times10^{-5}J$	0.75
B.6.a	$W_h = \int_{t_0}^{t_1} R\,i^2\,dt = W_h = \frac{RI_0^2\tau}{2}(e^0 - e^{-5}) = 7.15x10^{-5}J$	0.75
B.6.b	$W_e = W_h$ then the electric energy lost by the capacitor is transformed to heat energy through the resistor	0.5

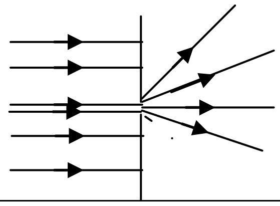
Second exercise (7.5 points)

Part of the Q	Answer	Mark
A.1	Connection 	0.5
A.2.a	In A.C. and in an RL circuit, u_G leads u_{DB} (on i_1). and a leads ⇒a gives u_{AB} Or $U_{m1} > U_{m\,(BD)}$ and Umg > $U_{m\,BD}$) ⇒a gives u_{AB}	0.5
A.2.b.i	$T = 8\times1 = 8$ ms = 0.008 S; $\omega = \frac{2\pi}{T} = 250\ \pi$ rd/s.	1.00
A.2.b.ii	$U_m = 3.5\times1 = 3.5$ V; $U_{m1} = 2.5\times1 = 2.5$ V.	1.00
A.2.b.iii	$\varphi = \frac{2\pi}{8}\times1 = \frac{\pi}{4}$ rad.	0.50
A.3.a.	$u_{R1} = 2.5\sin(250\pi t - \frac{\pi}{4})$.	0.50
A.3.b	$I_{m1} = \frac{U_{m1}}{R_1} = \frac{2.5}{25} = 0.1$ A. $i_1 = 0.1\sin(250\pi t - \frac{\pi}{4})$. Or $i_1 = \frac{u_{R1}}{R} = 0.1\sin(250\pi - \frac{\pi}{4})$	0.50
A.4	$u = u_L + u_{DB}$; with: $u_L = L\frac{di_1}{dt} = 25\pi L\cos(250\pi t - \frac{\pi}{4})$; $u_{DB} = R_1 i_1 = 2.5\sin(250\pi t - \frac{\pi}{4})$. We have then: $3.5\sin(250\pi t) = 25\pi L\cos(250\pi t - \frac{\pi}{4}) + 2.5\sin(250\pi t - \frac{\pi}{4})$. For t = 0, we have: $0 = 25\pi L\frac{\sqrt{2}}{2} - 2.5\frac{\sqrt{2}}{2} \Rightarrow \frac{\sqrt{2}}{2}\times L = \frac{0.1}{\pi} = 0.032$ H.	1.25
B.1	$u_c = u_{R1} = 2.5\sin(250\pi t - \frac{\pi}{4})$.	0.50
B.2.	$i_2 = C\frac{du_C}{dt} = 625\pi C\cos(250\pi t - \frac{\pi}{4})$	0.5
B.3	The ammeter gives $I_{eff} = 0.0277$A ⇒$I_{2M} = I_{eff}\sqrt{2} = 0.0391$A But $I_{2m} = 625C \Rightarrow C = \frac{I_{2m}}{625\pi} = 2\text{x}10^{-5}$ F	0.75

Third exercise (7.5 points)

Part of the Q	Answer	Mark
A.1.	M.E = $\frac{1}{2}I\theta'^2 + \frac{1}{2}C\theta^2$	1.00
A.2.	M.E = Cte $\Rightarrow \frac{dE_m}{dt} = 0 \Rightarrow I\theta'\theta'' + C\theta\theta' = 0 \Rightarrow \theta'' + \frac{C}{I}\theta = 0$	1.00
A.3.	$\theta = \theta_m \cos(\frac{2\pi}{T_0}t + \varphi) \Rightarrow \theta' = -\theta_m \frac{2\pi}{T_0}\sin(\frac{2\pi}{T_0}t + \varphi)$ $\Rightarrow \theta'' = -\theta_m (\frac{2\pi}{T_0})^2 \sin(\frac{2\pi}{T_0}t + \varphi) = -(\frac{2\pi}{T_0})^2\theta$ Sub. In the differential equation: $\frac{4\pi^2}{T_0^2}\theta + \frac{C}{I}\theta = 0$ $\Rightarrow \omega_0 = \sqrt{\frac{C}{I}} \Rightarrow T_0 = 2\pi\sqrt{\frac{I}{C}} \Rightarrow T_0 \approx 0.4$ s. $\theta = 0.1$rad $\Rightarrow \theta_m \cos\varphi = 0.1 \Rightarrow \varphi = 0$	1.5
B.1.a	$M.E_0 = \frac{1}{2}C\ \theta_{0m}^2 = 4\times10^{-6}$ J	0.75
B.1.b	$\theta_{0m} = 0.1$rad $\Rightarrow \theta_{1m} = \frac{0.1\times 97.5}{100} = 0.0975$ rad. $\Rightarrow \lvert\Delta E\rvert = \frac{1}{2}C\ (\theta_{0m}^2 - \theta_{1m}^2) = 1.97\times10^{-7}$ J	1.25
B.2	$P_{av} = \frac{\Delta E}{T} = -4.92\times10^{-7}$ W	0.75
C	The energy used for driving is : $\frac{0.8\times25}{100} = 0.2$ J. The duration of driving is : $t = \frac{0.2}{4.92\times10^{-7}} = 406504$ s; $t = \frac{406504}{24\times3600} = 4.7$ day	1.25

Fourth exercise : (7.5 points)

Part of the Q	Answer	Mark
A.1	The width of the slit a_1 is of the order of mm (or λ has to be of the same order of $a_1 (a_1 = 10^3 \lambda)$.	0.50
A.2.	Aspect of the emerging beam. 	0.50
A.3	We observe : • Alternate bright and dark fringes. • The direction of the diffraction pattern is perpendicular to that of the slit. • The width of the central bright fringe is twice as broad as others.	0.75
A.4	$\sin\alpha = \frac{2\lambda}{a_1}$ and in case of small angles $\sin\alpha \approx \alpha_{rd} \Rightarrow \alpha = \frac{2\lambda}{a_1}$	0.50
A.5.a	Figure $\tan\frac{\alpha}{2} = \frac{L}{2D}$ and case of small angles $\tan\alpha \approx \alpha_{rd} \Rightarrow L = \alpha \times D = \frac{2\lambda D}{a_1}$.	0.75
A.5.b	$L = \frac{2\times 0.633\times 10^{-3}\times 2\times 10^{3}}{0.1} mm = 25\ mm$.	0.50
A.6.	The displacement of 1 mm is due to the distance a =1 mm between the two slits	0.50
B.1.	$\delta = \frac{ax}{D}$, Dark fringe $\delta=(2k+1)\frac{\lambda}{2} \Rightarrow x = (2k+1)\frac{\lambda D}{2a}$	0.75
B.2.	$i = x_{k+1} - x_k = \frac{[2(k+1)+1]\lambda D}{2a} - \frac{(2k+1)\lambda D}{2a} = \frac{\lambda D}{a}$.	0.75
B.3.	$i = \frac{0.6\times 10^{-3}\times 2\times 10^{3}}{1} = 1.2\ mm.$	0.50
B.4.	$\frac{x}{i} = \frac{2.4}{1.2} = 2 \Rightarrow x = 2i \Rightarrow$ center of the second bright fringe	0.75
B.5.	$x=(2k+1)\frac{\lambda D}{2a} \Rightarrow 2.4\times 10^{-3} = (2k+1)\frac{600\times 10^{-9}\times 2}{2\times 10^{-3}}$ $\Rightarrow k = 2$ then it corresponds to the center of third dark fringe	0.75