مسابقة في مادة الفيزياء	الاسم:
المدة ثلاث ساعات	الرقم:

This exam is formed of four exercises in four pages numbered from 1 to 4.
The use of non-programmable calculator is recommended,

<u>First exercise</u>: (7.5 points) Variation of the kinetic energy of a system

The aim of this exercise is to verify the theorem of kinetic energy of a system.
The skier (S) of mass M = 80 kg, moves down from O to A, with a constant velocity $\vec{v} = v\vec{i}$, where v = 30m/s along the line of greatest slope of a track inclined by an angle $\alpha = 30°$ with the horizontal. The track exerts on the skier a constant force of friction $\vec{f} = -f\vec{i}$.
The motion of the skier is represented by the motion of its center of mass G on $\overrightarrow{x'x}$ where $\vec{i}$ is a unit vector along this axis (figure 1).
Neglect the air resistance on the skier.
Take:

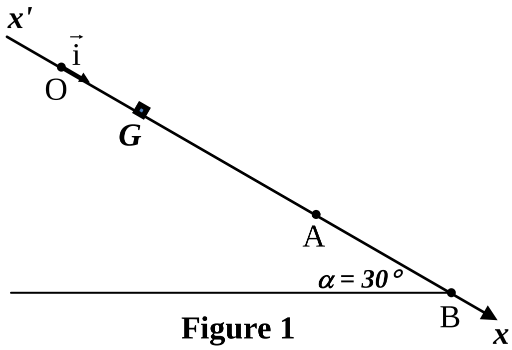

Figure 1

- the horizontal plane through B as a gravitational potential energy reference for the system (skier, Earth).
- $g = 10\ m/s^2$.

1) Name and represent the external forces acting on G along the path OA.

2) **a)** Show that the linear momentum $\vec{P}$ of the skier is constant.
b) Apply Newton's second law on the skier, between the points O and A, deduce the magnitude of $\vec{f}$.

3) The skier, upon reaching A, starts exerting a constant braking force $\vec{f_1} = -f_1\vec{i}$ to stop at B. The skier covers the distance AB during a time interval Δt = 3 s.
a) Determine the magnitude of $\vec{f_1}$, assuming that $\frac{\Delta\vec{P}}{\Delta t} \approx \frac{d\vec{P}}{dt}$.
b) The mechanical energy of the system (skier, Earth) decreases from A to B. Name the forces that are responsible of this decrease.
c) Determine the distance AB covered by the skier during the time interval Δt.

4) **a)** Determine between A and B :
i. the variation of the gravitational potential energy ΔPE_g of the system (skier, Earth) ;
ii. the work done by the weight $W_{m\vec{g}}$.
b) Compare ΔPE_g and $W_{m\vec{g}}$.

5) ΔKE and $\sum W_{\vec{F}_{ext}}$ are respectively the variation of the kinetic energy of the skier and the algebraic sum of the work done by the external forces between A and B.
Verify, between A and B, the work-energy theorem: $\Delta KE = \sum W_{\vec{F}_{ext}}$.

<u>Second exercise</u>: (7.5 points) The characteristics of RLC series circuit

Consider:

- a generator G delivering an alternating sinusoidal voltage :
  $u_{AM} = u_G = u = U\sqrt{2}\cos\omega t$ (u in V and t in s), where U = 5 V and $\omega = 2\pi f$ with adjustable frequency f;
- a coil of inductance L and of negligible resistance;
- a capacitor of capacitance C;
- a resistor of resistance R = 150 Ω;
- an oscilloscope;
- a milli-ammeter of negligible resistance;
- a switch K and connecting wires.

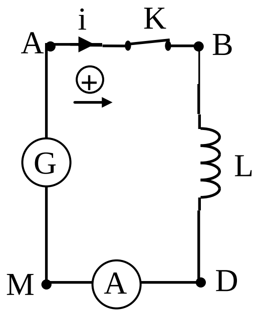

Figure 1

In order to determine L and C, we perform the following experiments:

A- First experiment

We perform successively the setup of figure 1 and of figure 2.

For f = 500 Hz, the effective current I, indicated by the milli-ammeter, has the same value I= 50mA in both setups. Take $\frac{1}{\pi} = 0.32$.

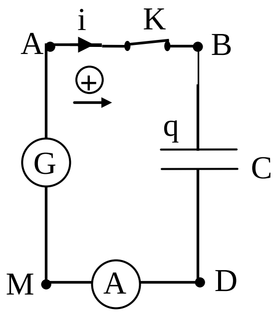

Figure 2

1) The coil is connected across the terminals of G (figure 1). The circuit carries a current i of expression $I = I\sqrt{2}\cos(\omega t - \frac{\pi}{2})$. (i in A and t in s)

a) Determine the expression of the voltage $u_{BD} = u_{coil}$ in terms of L, ω, I and t.

b) Deduce the value of L.

2) The capacitor is connected across the terminals of G (figure 2). The circuit carries a current i of expression $i = I\sqrt{2}\cos(\omega t + \frac{\pi}{2})$.

a) Determine the expression of the voltage $u_{BD} = u_C$ in terms of C, ω, I and t.

b) Deduce the value of C.

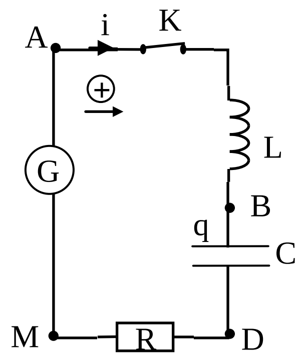

Figure 3

B- Second experiment

To verify the values obtained for L and C in the first experiment, we perform the setup of the circuit shown in Figure 3. This circuit contains the generator, the coil, the capacitor, and the resistor of resistance R = 150 Ω. The oscilloscope, displays on channel (1), the voltage u_{AM} across the generator, and on channel (2), the voltage u_{DM} across the resistor. Figure (4) shows the waveforms representing u_{AM} and u_{DM}. The circuit carries a current $i = I\sqrt{2}\cos(\omega t + \varphi)$.

1) Redraw figure 3 and indicate the connections of the oscilloscope.

2) Apply the law of addition of voltages and give t a particular value, show that: $\tan\varphi = \dfrac{\frac{1}{C\omega} - L\omega}{R}$.

3) Referring to the waveform of Figure 4 observed on the screen of the oscilloscope, determine:

a) the frequency f;

b) the phase difference φ between u and i.

4) The effective voltage U being kept constant and we vary f. We observe that u_{AM} and u_{DM} become in phase when f takes the value f_0 = 500 Hz.

a) Name the phenomenon that takes place.

b) Give the relation giving ω_0 in terms of L and C.

5) Determine L and C.

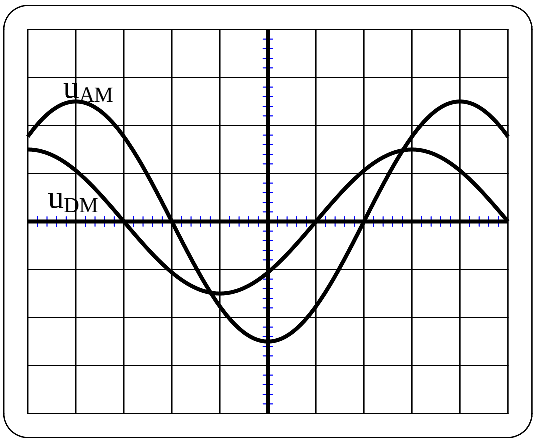

Figure 4

Horizontal Sensitivity : 0.5 ms/div

Third exercise: (7.5 points) Corpuscular aspect of light

The aim of this exercise is to study the emission spectrum of the hydrogen atom and use the emitted light to produce photoelectric effect.

Given:

- Planck's constant: $h = 6.62 \times 10^{-34}$ J.s ;
- Speed of light in vacuum: $c = 3 \times 10^{8}$ m/s ;
- 1 eV = 1.602×10^{-19} J ;
- Elementary charge: $e = 1.602 \times 10^{-19}$ C ;
- 1 nm = 10^{-9} m.

A. Hydrogen atom

The emission spectrum of the hydrogen atom constituted in its visible part of four radiations denoted by H_α, H_β, H_γ and H_δ of respective wavelengths, in vacuum, 656.27nm, 486.13nm, 435.05nm and 410.17nm.

I. In 1885, Balmer noticed that the wavelengths λ of these four radiations verify the empirical formula $\lambda = \lambda_0 \frac{n^2}{n^2 - 4}$ where λ_0 = 364.6 nm where n is a non-zero positive whole number.

1) The smallest value of n is 3. Justify.

2) Calculate the wavelength corresponding to this radiation.

3) Deduce the values of n corresponding to the wavelengths of the other three visible radiations in the emission spectrum of the hydrogen atom.

II. The quantized energy levels of the hydrogen atom are given by the formula:

$E_n = -\frac{13.6}{n^2}$ (in eV) where n is a whole non-zero positive number.

Using the expression of E_n, determine the energy of the atom when it is:

1) in the ground state.

2) in each of the first five excited levels.

3) ionized state.

B. Photoelectric effect

A hydrogen lamp of power P_S = 2W, emits uniformly radiation in all directions in a homogeneous and non- absorbing medium. This lamp illuminates a potassium cathode C of a photoelectric cell of work functionW_0 = 2.20 eV and of a surface area s = 2cm^2 placed at a distance D = 1.25m from the lamp (figure1).

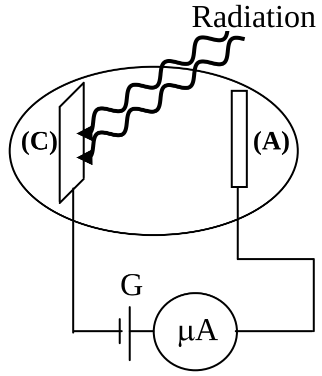

Figure 1

1) Calculate the threshold wavelength of the potassium cathode.

2) Among the rays of Balmer series, specify the radiation that can produce photoelectric emission.

3) Using a filter we illuminate the cell by a blue light H_β of wavelength λ = 486.13nm. The generator G is adjusted so that the anode (A) captures all the emitted electrons by the cathode of quantum efficiency r = 0.875%.

a) Show that the received power of the radiation P_0 of the cell is 2.04×10^{-5}W.

b) Determine the number N_0 of the incident photons received by the cathode C in one second.

c) Determine the current in the circuit.

Fourth exercise: (7.5 points) Compound Pendulum

The aim of this exercise is to study the motion of a compound pendulum.

Consider a compound pendulum (P) consists:

- of a straight and homogeneous rod (R) of length AB = ℓ and of mass m ;
- of a solid (S), taken as a particle of mass m_1, free to slide along the part OB of the rod, O being the midpoint of the rod.

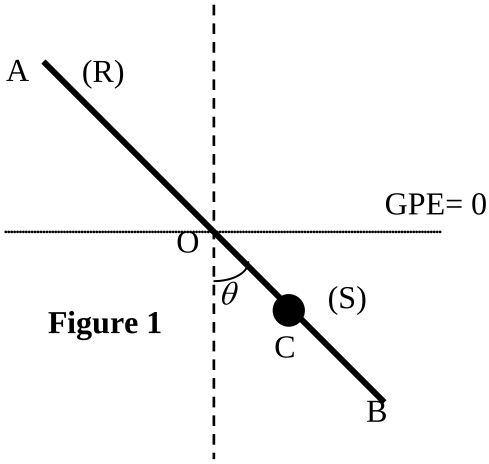

Figure 1

We fix (S) at a point C such that $\overline{OC}$ = x (x > 0).

(P) can oscillate, in a vertical plane, around a horizontal axis (Δ) perpendicular to the rod at O (figure 1).

(P) is shifted from its equilibrium position by a small angle θ_m then released without initial velocity at the instant $t_0 = 0$, the pendulum oscillate then, without friction, around its equilibrium position.

At the instant t, the angular elongation of the pendulum is θ and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

Given: moment of inertia of the rod about the axis of rotation (Δ): $I_0 = \frac{1}{12} m \ell^2$, $m = 3m_1$,

$\ell = 0.5$ m, g = 10 m/s^2 and $\pi^2 = 10$.

For small θ: $\cos\theta \approx 1 - \frac{\theta^2}{2}$ and $\sin\theta \approx \theta$ (θ in rd).

G is the center of inertia of the pendulum and the horizontal plane passing through O is taken as reference level of the gravitational potential energy.

1) Show that:

a) $\overline{OG} = \frac{x}{4}$;

b) The expression of the moment of inertia of the pendulum is: $I = \frac{m}{12}(\ell^2 + 4x^2)$.

2) Determine the expression the mechanical energy of the system (pendulum, Earth) in terms of θ, θ', m ,x and ℓ.

3) **a)** Establish the second order differential equation in θ which governs the oscillations of the pendulum.

b) Deduce that the expression of the proper period of the pendulum is: $T_0 = \sqrt{\frac{4x^2 + \ell^2}{x}}$.

4) **a)** Determine the value of x for which T_0 is minimum.

b) Deduce that $T_{0(min)} = 1.41$ s.

5) Using a coupling device, the pendulum (P) plays the role of an exciter for a simple pendulum (P_1) of length $\ell_1 = 65$ cm. The oscillations of (P) and (P_1) are slightly damped.

a) Knowing that the proper period of the simple pendulum, for small oscillation, is $T = 2\pi\sqrt{\frac{\ell}{g}}$,

Calculate the value of the proper period T_{01} of (P_1).

b) **i)** (P) oscillates now with its minimum period. It is noticed that (P_1) does not enter in amplitude resonance with (P). Justify.

ii) We move (S) between O and B. For a value x_0 of x, we notice that (P_1) oscillates with large amplitude. Calculate the value of x_0.

<table>
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الاثنين ١٣ حزيران ٢٠١٦</td><td>امتحانات الشهادة الثانوية العامة
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المديرية العامة للتربية
دائرة الامتحانات</td></tr>
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المدة ثلاث ساعات</td><td>مشروع معيار التصحيح</td></tr>
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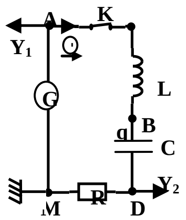
First exercise (7.5 points)

Part of the Q	Answer	Mark
1	The forces acting on the skier : • Normal reaction $\vec{N}$; • Weight $m\vec{g}$; • The frictional force $\vec{f}$ Diagram.	3/4
2.a	$\vec{P} = M\vec{V}$ since $\vec{V} = \overrightarrow{Cte} \Rightarrow \vec{P} = \overrightarrow{Cte}$.	3/4
2.b	$\frac{d\vec{P}}{dt} = \overrightarrow{Mg} + \vec{N} + \vec{f} = \vec{0}$ project along x'x: Mgsinα – f = 0 $\Rightarrow$ f = Mgsinα = 400 N.	1
3.a	$\frac{d\vec{P}}{dt} = \overrightarrow{Mg} + \vec{f} + \vec{N} + \vec{f_1} = \frac{\Delta\vec{P}}{\Delta t}$ Project along x'x $\Rightarrow -f_1 = \frac{MV_B - MV_A}{\Delta t} = -\frac{MV_A}{\Delta t} \Rightarrow f_1$= 800 N. **Or :** $\frac{\Delta\vec{P}}{\Delta t} = \sum\vec{F}_{ext} \Rightarrow \frac{\vec{P}_O - \vec{P}_A}{\Delta t} = \sum\vec{F}_{ext}$ Project along x'x : $\frac{0 - MV_A}{\Delta t} = Mg\sin\alpha - f - f_1 = 0 - f_1 = -f_1 \Rightarrow f_1 = 800N$	1
3.b	Because friction and braking forces	1/2
3.c	ΔM.E = W($\vec{f}$) + W($\vec{f_1}$) $\Rightarrow$ $M.E_B - M.E_A$ = W($\vec{f}$) + W($\vec{f_1}$) $\Rightarrow$ -1/2 MV^2 –Mg AB sinα = - f .AB – f_1 . AB $\Rightarrow$ (40×900) + (400 × AB) = 1200 × AB $\Rightarrow$ AB = 45 m.	1
4.a.i	$\Delta GPE = GPE_B - GPE_A = 0 - Mg\ AB\sin\alpha$ = - Mg AB sinα = - 1800 J	3/4
4.a.ii	W($M\vec{g}$) = Mgh = Mg AB sinα = 1800 J	1/2
4.b	Δ(GPE) = - W($M\vec{g}$).	1/4
5	ΔM.E = ΔK.E + ΔGP.E = W($\vec{f}$) + W($\vec{f_1}$) $\Rightarrow$ΔK.E = W($M\vec{g}$)+ W($\vec{f}$) + W($\vec{f_1}$) since W($\vec{N}$) =0 $\Rightarrow$ ΔK.E = $\sum W_{\vec{F}_{ext}}$ **Or :** ΔM.E = ΔK.E + ΔGP.E = W($\vec{f}$) + W($\vec{f_1}$) $\Rightarrow$ΔK.E. = W($\vec{f}$) + W($\vec{f_1}$) – ΔGP.E = W($\vec{f}$) + W($\vec{f_1}$) + W($M\vec{g}$) Or W($\vec{N}$) =0 $\Rightarrow$ Δ K.E. = $\sum W_{\vec{F}_{ext}}$	1

Second exercise (7.5 points)

Part of the Q	Answer	Mark
A.1.a	$u_{BD} = u_L = L\frac{di}{dt} = -LI\omega\sqrt{2}\sin(\omega t - \frac{\pi}{2})$	¾
A.1.b	$u_{AM} = u_{BD} \Rightarrow -LI\omega\sqrt{2}\sin(\omega t - \frac{\pi}{2}) = U\sqrt{2}\cos\omega t \Rightarrow LI\omega\sqrt{2}\cos(\frac{\pi}{2} + \omega t - \frac{\pi}{2}) = U\sqrt{2}\cos\omega t$ By comparison: $U\sqrt{2} = LI\omega\sqrt{2} \Rightarrow L = 0.032$ H = 32 mH. **Or**: $-LI\omega\sqrt{2}\sin(\omega t - \frac{\pi}{2}) = U\sqrt{2}\cos\omega t$ For t = 0: $U\sqrt{2} = LI\omega\sqrt{2} \Rightarrow L = 0.032$ H = 32 mH.	¾
A.2.a	$i = C\frac{du_C}{dt} \Rightarrow u_C = \frac{1}{C}\int i dt = \frac{I\sqrt{2}}{C\omega}\sin(\omega t + \varphi)$	¾
A.2.b	$u_{AM} = u_{BD} \Rightarrow U\sqrt{2}\cos\omega t = \frac{I\sqrt{2}}{C\omega}\sin(\omega t + \frac{\pi}{2}) \Rightarrow U\sqrt{2}\cos\omega t = \frac{I\sqrt{2}}{C\omega}\cos(\frac{\pi}{2} - \omega t - \frac{\pi}{2})$ By comparison : $U\sqrt{2} = \frac{I\sqrt{2}}{C\omega} \Rightarrow C = 3.2\times10^{-6}$ F = 3.2 μF **Or** : $u_{AM} = u_{BD} \Rightarrow U\sqrt{2}\cos\omega t = \frac{I\sqrt{2}}{C\omega}\sin(\omega t + \frac{\pi}{2})$ For t = 0 : $U\sqrt{2} = \frac{I\sqrt{2}}{C\omega}\sin(\frac{\pi}{2}) \Rightarrow C = 3.2\times10^{-6}$ F = 3.2 μF	¾
B.1	Connections of the oscilloscope  	¼
B.2	$u_{AM} = u_{AB} + u_{BD} + u_{DM} \Rightarrow$ $U\sqrt{2}\cos\omega t = -LI\omega\sqrt{2}\sin(\omega t + \varphi) + \frac{I\sqrt{2}}{C\omega}\sin(\omega t + \varphi) + RI\sqrt{2}\cos(\omega t + \varphi)$ For $\omega t = \frac{\pi}{2} \Rightarrow 0 = -LI\sqrt{2}\cos\varphi + \frac{I\sqrt{2}}{C\omega}\cos\varphi - RI\sqrt{2}\sin\varphi \Rightarrow \tan\varphi = \frac{\frac{1}{C\omega} - L\omega}{R}$	**1**
B.3.a	$T = 4$ ms $\Rightarrow f = \frac{1}{T} = 250$ Hz .	½
B.3.b	$\lvert\varphi\rvert = \frac{2\pi}{8} = \frac{\pi}{4}$ rad.	½
B.4.a	Current resonance	¼
B.4.b	$\varphi = 0 \Rightarrow \tan\varphi = 0 \Rightarrow \frac{1}{C\omega_0} = L\omega_0 \Rightarrow LC = \frac{1}{\omega_0^2}$.	½
B.5	$\varphi = \frac{\pi}{4} \Rightarrow 1 = \frac{\frac{1}{C\omega} - L\omega}{R} \Rightarrow C = \frac{1 - LC\omega^2}{R\omega} = 3.2\times10^{-6}$ F = 3.2 μF $\Rightarrow LC = \frac{1}{\omega_0^2} \Rightarrow L = \frac{1}{C\omega_0^2} = 32$ mH	**1 ½**

Third exercise (7.5 points)

Part of the Q	Answer	Mark
A.I.1	λ , λ_0 and n^2 are positive $\Rightarrow n^2 - 4 > 0 \Rightarrow n > 2 \Rightarrow$ the smallest value is n = 3.	½
A.I.2	$\lambda = \lambda_0 \frac{n^2}{n^2 - 4} \Rightarrow \lambda = 656.46$ nm.	½
A.I.3	In these conditions: n = 4 gives $\lambda = 486.13$ nm n = 5 gives $\lambda = 435.05$ nm n = 6 gives $\lambda = 410.17$ nm	¾
A.II.1	Ground state n = 1: $E_1 = -13.6$ eV.	½
A.II.2	1st energy level (excited): n = 2: $E_2 = -\frac{13.6}{2^2} = -3.4\text{eV}$ $E_3 = -1.51$ eV; $E_4 = -0.85$ eV; $E_5 = -0.54$ eV and $E_6 = -0.38$ eV	**1 ¼**
A.II.3	The atom is ionized when $n \to \infty \Rightarrow E_\infty = 0$	½
B.1	$W_0 = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{W_0} = 5.65\times10^{-7}\text{m} = 565\text{nm}$	¾
B.2	The radiations of Balmer series that can produce photoelectric emission verifies the relation $\lambda \langle \lambda_0$; H_β, H_γ and H_δ produce this emission because $\lambda \langle \lambda_0$	½
B.3.a	$P_0 = \frac{P_S \times s}{4\pi D^2} = 2.04\times10^{-5}\text{W}$	¾
B.3.b	$N_{0/s} = \frac{P_0}{E_{photon}} = \frac{P_0 \times \lambda_\beta}{h \times c} = 4.99\times10^{13}\text{photons/s}$	¾
B.3.b	The number of effective photons = number of emitted electrons N_e $\Rightarrow N_e = r \times N_0 = 4.37\times10^{11}$ electrons/s $I_0 = \frac{q}{t} = \frac{N_e e}{t} = 6.99\times10^{-8}\text{A}$	¾

Fourth exercise (7.5 points)

Part of the Q	Answer	Mark
1.a	$(m + m_1)\overrightarrow{OG} = m\,\overrightarrow{OO} + m_1\overrightarrow{OM} \Rightarrow \overline{OG} = \frac{x}{4}$	½
1.b	$I_{(sys)} = I_{(rod)} + I_{(S)} \Rightarrow I = \frac{1}{12}m\ell^2 + \frac{m}{3}x^2 = \frac{m}{12}(4x^2+\ell^2)$	½
2	$ME = \frac{1}{2}I\theta'^2 - (m+m_1)gOG\cos\theta = \frac{m}{24}(4x^2+\ell^2)\theta'^2 - \frac{m}{3}gx\cos\theta$	¾
3.a	$ME = Cte \Rightarrow \frac{dME}{dt} = 0 \Rightarrow \frac{m}{12}(4x^2+\ell^2)\theta'\theta'' + \frac{m}{3}gx\theta'\sin\theta = 0$, $\theta' \neq 0$ For small angle $\sin\theta \approx \theta$ (rd) $\Rightarrow \theta'' + \left(\frac{4gx}{4x^2+\ell^2}\right)\theta = 0$.	¾
3.b	This differential equation has the form: $\theta'' + \omega_0^2\theta = 0 \Rightarrow \omega_0 = \sqrt{\frac{4gx}{4x^2+\ell^2}} \Rightarrow T_0 = \sqrt{\frac{4x^2+\ell^2}{x}}$.	½
4.a	$\frac{dT_0}{dx} = \frac{1}{2}\left(\frac{4x^2-\ell^2}{x^2}\right)\left(\frac{4x^2+\ell^2}{x}\right)^{-\frac{1}{2}}$; T_0 is minimum when $\frac{dT_0}{dx} = 0$ for x $\in \left]0,\frac{\ell}{2}\right] \Rightarrow 4x^2 - \ell^2 = 0$; then T_0 is minimal for $4x^2 = \ell^2 \Rightarrow x = \frac{\ell}{2}$.	**1 ½**
4.b	$T_0 = \sqrt{\frac{\ell^2+\ell^2}{\frac{\ell}{2}}} = 2\sqrt{\ell} = 1.41$ s	½
5.a	$T_{01} = 2\pi\sqrt{\frac{\ell_1}{g}} = 1.61$ s	½
5.b.i	The phenomenon of amplitude resonance will take place when the proper period of the exciter becomes equal (very close) of that of the resonator. As $T_0 = 1.41$ s of (P) is smaller than $T_{01} = 1.61$ S of (P_1), therefore the phenomenon of resonance does not take place	½
5.b.ii	(P_1) oscillates with large amplitude, therefore it is in resonance of amplitude with (P); and then the proper period of (P) is equal to T_{01}. = 1.61 s. $4x^2 - (1{,}61)^2x + \ell^2 = 0$ $\Rightarrow$ The solution of this quadratic equation gives; x_1 = 53 cm (rejected because it is > than $\frac{\ell}{2}$ = 25 cm) and x_2 = 11.75 cm (accepted)	**1 ½**