عدد المسائل: ست	**مسابقة في مادة الرياضيات** **المدة: أربع ساعات**	**الاسم:** **الرقم:**

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

مسابقة في مادة الرياضيات

المدة: أربع ساعات

(باللغة الإنكليزية)

الاسم:..

الرقم:..

I- (2 points)

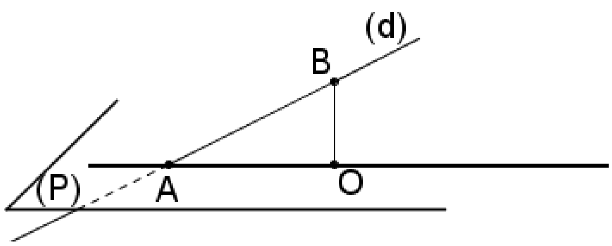
In the table below, **only one** of the proposed answers to each question **is correct.**

Write down the number of each question and give, **with justification**, the answer corresponding to it.

Nº	Questions	Answers a	Answers b
1	z is a complex number. One of the roots of the equation $z^4 + z^2 - i\sqrt{3} = 0$ is	$e^{i\frac{\pi}{6}}$	$e^{-i\frac{\pi}{6}}$
2	Denote by f the function defined on $\mathbb{R}$ as $f(x) = \dfrac{1}{x^2 + 4x + 8}$. An antiderivative of f is	$\frac{1}{2}\arctan\left(\frac{x+2}{2}\right)$	$\frac{1}{4}\arctan\left(\frac{x+2}{2}\right)$
3	m is a real number (m > 1). If $J = \int_m^{m+1} \frac{1}{x}\,dx$, then J belongs to the interval	$[m , m+1]$	$\left[\frac{1}{m+1}, \frac{1}{m}\right]$
4	z is a complex number. If $z = 1 + \cos\theta + i\sin\theta$, with $\pi < \theta < 2\pi$, then $\lvert z \rvert =$	$2\cos\left(\frac{\theta}{2}\right)$	$-2\cos\left(\frac{\theta}{2}\right)$

II- (2 points)

In the space referred to an orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the plane (P) with **equation x + z = 0** and the line (d) with **parametric equations** $\begin{cases} x = -t + 1 \\ y = -2t \\ z = 3t + 1 \end{cases}$ (t ∈ $\mathbb{R}$).



1) **Calculate** the coordinates of point A, **the intersection of (d) and (P).**

2) The point B(1 , 0 , 1) is **on (d).**

a- **Prove that** O is the orthogonal projection of B on (P).

b- **Deduce** a system of parametric equations of the line (Δ), the orthogonal projection of (d) on (P).

3) The point J(–5 , 2 , 5) is on (P).

Calculate the volume of the tetrahedron **OBJA**.

4) In the plane (P), consider the hyperbola (H) with **foci O and A** and eccentricity e = 3.

a- **Verify** that the point I(1 , 1 , –1) is the center of (H).

b- **Verify that** the coordinates of S and G, are S $(\frac{4}{3}, \frac{4}{3}, \frac{-4}{3})$ and G $(\frac{2}{3}, \frac{2}{3}, \frac{-2}{3})$

III- (3 points)

Consider two fair cubic dice.

The faces of each die are numbered from 1 to 6.

The two dice are rolled.

Denote by X the random variable that is defined as follows:

- if the two numbers shown on the dice **are different**, then **X is equal to the greater between them**;
- if the two numbers shown on the dice **are equal**, then **X is equal to one them.**

For example, – if the two numbers shown on the dice are 2 and 3, then X = 3 – if the two numbers shown on the dice are 4 and 4, then X = 4

1) a- **Calculate** the probabilities $P(X = 1)$ and $P(X = 2)$.

b- **Prove** that $P(X \le 3) = \frac{1}{4}$.

2) In this part, consider an urn U that contains **6 balls: 4 red and 2 blue**.

The two dice are rolled:

- if $X \le 3$, then 3 balls are **randomly and simultaneously** selected from U;
- if $X > 3$, then 3 balls are selected **randomly and successively with replacement** from U.

Consider the following events:

A: "$X \le 3$"

S: "The 3 selected balls have the **same color**"

a- **Calculate**:

- $P\left(S/A\right)$
- $P\left(A \cap S\right)$.

b- **Verify** that $P\left(\overline{A} \cap S\right) = \frac{1}{4}$

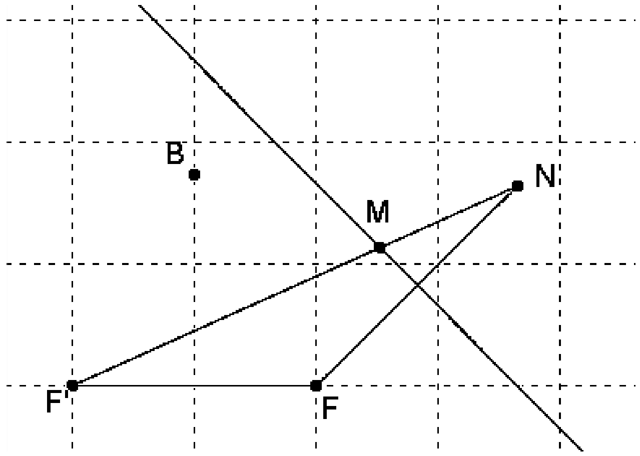
Calculate $P\left(S\right)$.

c- Knowing that $X > 3$, **calculate** the probability that the three selected balls do not have the same color.

IV- (3 points)

In the below figure,

- F and F' are two fixed points so that **FF' = 2.**
- N is a variable point on the circle with center F' and radius 4.
- The perpendicular bisector of [NF] intersects [F'N] at M.
- B is a fixed point so that BF'F is an equilateral triangle.



1) a- **Show that** MF + MF' = 4.

b- **Deduce that** M moves on a conic (E) whose nature, foci and center O are to be determined.

2) Let A be the symmetric of O with respect to F.

a- **Show that** A is <u>one of the vertices</u> of (E).

b- **Verify that (OB)** is the <u>non-focal axis</u> of (E)

Verify that B is <u>one of the vertices</u> of (E).

c- **Draw** (E).

3) The plane is referred to a direct orthonormal system $\left(O;\vec{i},\vec{j}\right)$ with $\vec{i}=\overrightarrow{OF}$.

a- **Verify that** $\frac{x^2}{4}+\frac{y^2}{3}=1$ is an equation of (E).

b- **Write** an equation of (d) <u>the directrix</u> of (E) associated to F.

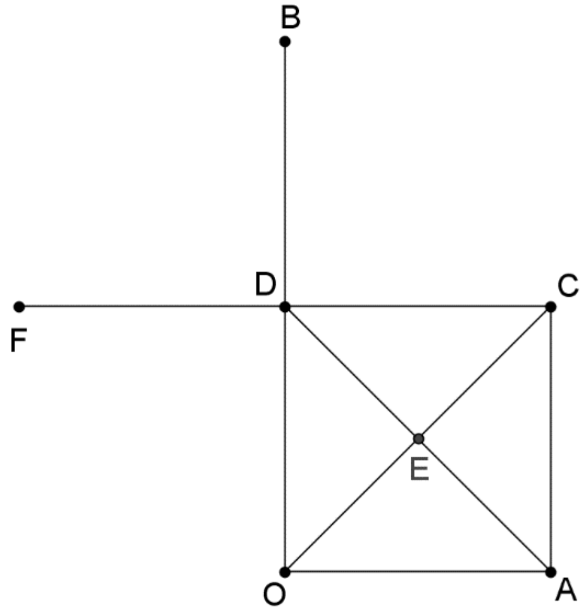
c- Let L(α,β) be a point on (E) where α and β are two real numbers with $\beta \neq 0$.

- **Verify that** the equation of <u>the tangent (T)</u> to (E) at L is $\frac{\propto x}{4}+\frac{\beta y}{3}=1$.

V- (3 points)

In the figure below;

- OACD is a direct square with center E and side 2.
- F is the symmetric of C with respect to D.
- B is the symmetric of O with respect to D.



Denote by S the direct plane similitude of center O that maps A onto B.

Part A

1)a- **Calculate** the ratio k and an angle α of S.

b- **Verify** that S(E) = F.

c- **Prove** that the triangle OBF is right isosceles.

2)Consider the direct plane similitude $S'\left(E, 2, \frac{\pi}{2}\right)$ and the transformation $h = S \circ S'$.

Denote by W the center of h. **Show that** $\overrightarrow{WF} = -4\,\overrightarrow{WF}$.

Part B

The plane is referred to a direct orthonormal system $\left(O; \vec{u}, \vec{v}\right)$ with

$\vec{u} = \frac{1}{2}\overrightarrow{OA}.$

1) **Show that** the complex form of h is $z' = -4z + 2 + 6i$ and **deduce** the affix of W.

2) For all $n \in \mathbb{N}$, consider the numerical sequence (d_n) defined as $d_n = WE_n$ where $E_0 = E$ and $E_{n+1} = h(E_n)$.

   a- **Verify that** $d_0 = \frac{\sqrt{10}}{5}$.

   b- **Show that** (d_n) is a geometric sequence of common ratio 4.

   c- **Determine** the number of points E_n such that $d_n < 2019$.

VI- (7 points)

Part A

Consider the differential equation (E): $y'' - 2y' + y = e^{2x}$ and let $y = z + e^{2x}$.

1) **Write** a <u>differential equation</u> (E') satisfied by z.

2) **Determine** the <u>general solution</u> of (E').

   Determine the <u>general solution</u> of (E).

3) **Determine** the <u>particular solution</u> of (E) whose representative curve, in an orthonormal system, has at the point A(0 , –2) a tangent parallel to the x-axis.

Part B

Let f be the function defined on ℝ as $f(x) = e^{2x} + (x-3)e^{x}$.

Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) **Determine** $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to +\infty} \frac{f(x)}{x}$.

2) **Determine** $\lim\limits_{x \to -\infty} f(x)$ and **deduce** an asymptote to (C).

3) Let g be the function defined on ℝ as $g(x) = x - 2 + 2e^{x}$.

   a- **Set up** the table of variations of the function g.

   b- **Calculate** $g(0)$ then **deduce**, according to the values of x, the sign of $g(x)$.

4) **Verify that** $f'(x) = e^{x}g(x)$ and set up the table of variations of the function f.

5) **Show that** the equation $f(x) = 0$ has, on ℝ, a unique root α. **Verify** that $0.7 < \alpha < 0.8$.

6)**Draw** the curve (C).

7)a- **Prove that** f has, over [0 , + ∞[, an inverse function f^{-1} and **determine** its domain of definition.

b**- Draw** <u>the representative curve **(C')**</u> of f^{-1} in the same system $(O;\vec{i},\vec{j})$.

c- **Calculate**, in terms of α, the area of the region bounded by **(C')**, the x-axis and the y-axis.

Part C

Let h be the function given by as $h(x) = \arcsin\left(f^{-1}(x)\right)$.

Show that <u>the domain of definition</u> of the function h is $[-2, e^2 - 2e]$.