

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- [4 points]

A factory produces a certain article. The table below shows the supply y of this product, expressed in thousands of articles, in terms of the unit price x in thousands LL.

The sale price x_i in thousands LL.	2	3	5	7	10
Supply y_i in thousands of articles	5	7	8	9	11

The pattern remains valid for a price 20 000 LL

- Determine an equation of the regression line ($D_{y/x}$).
- Estimate the supplied number of articles for a unit price of 15 000 LL.
- The demand is modeled by the line ($D_{z/x}$) with equation $z = -0.567x + 8.266$.
The two lines ($D_{y/x}$) and ($D_{z/x}$) intersect at the $I(3.158 ; 6.474)$.
Give an economical interpretation of the coordinates of this point.
- Calculate, in LL, the revenue for a unit price of 7000 LL.
- Let $e(x)$ be the elasticity of demand in terms of the unit price x .
a) Show that $e(x) = \frac{567x}{-567x+8266}$.
b) Calculate $e(4)$. Give an economical interpretation of the value thus obtained.

II- [4 points]

Part A

Nisrine deposits 4 000 000 LL in a bank at an annual interest rate of 0.5% compounded yearly.

After compounding the interest, Nisrine adds yearly 200 000 LL to her account.

Denote by V_n the amount, in millions LL, that Nisrine has in her account after n years (n is a natural number).

Thus $V_0 = 4$.

- Justify, for all natural numbers n , that $V_{n+1} = 1.005V_n + 0.2$.
- Denote by (W_n) the sequence defined, for all natural numbers n , as $W_n = V_n + \alpha$.
a) Calculate α if (W_n) is a geometric sequence with common ratio 1.005.
b) Take $\alpha = 40$. Express W_n in terms of n and deduce that $V_n = 44(1.005)^n - 40$.
- a) Calculate the amount in Nisrine's account after ten years.
b) Calculate the amount of interest gained by Nisrine over ten years.
- Prove that the sequence (V_n) is increasing.

Part B

At the same date when Nisrine deposited the 4 000 000 LL, Nadia invested 8 000 000 LL in the same bank following the rule: $U_{n+1} = 1.005U_n$, where U_n is the amount, in millions LL, in Nadia's account after n years (n is a natural number). Thus $U_0 = 8$.

- Prove that (U_n) is a geometric sequence and that $U_n = 8(1.005)^n$.
- We admit that (U_n) is also an increasing sequence.
After how many years would the amount in Nisrine's account exceed the amount of money in Nadia's account for the first time?

III- [4 points]

An urn U contains 4 white balls, 5 red balls and 3 green balls.

Part A

We select randomly and successively 3 balls from U without replacement.

- 1) Calculate the probability of selecting three balls of the same color.
- 2) Prove that the probability of selecting at least one green ball and at least one red ball among the three selected balls is $\frac{21}{44}$.

Part B

A player pays an amount of 10 000 LL to participate in a game.

This game runs as follows:

The player selects at random and simultaneously two balls from the urn U.

- If at most one of the two selected balls is green, the player receives 5 000 LL and the game ends.
- If the two selected balls are both green, they are kept outside the urn U and the player receives 8 000 LL. After that, the player selects randomly and simultaneously two balls from the remaining 10 balls in U.
 - If these two selected balls are of the same color the player receives 12 000 LL and the game ends;
 - Otherwise, the player receives 2 000 LL and the game ends.

Let X be the random variable equal to the algebraic gain of the player

(The algebraic gain could be zero, positive or negative).

- 1) Justify that the values of X are: -5 000 ; 0 and 10 000.
- 2) Prove that $P(X = 0) = \frac{29}{990}$.
- 3) Determine the probability distribution of X.
- 4) Do you expect this player to win? Justify.

IV- [8 points]

Let f be the function defined on $[0; +\infty[$ as $f(x) = (-2x - 1)e^{-x} + 2$ and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

Part A

- 1) Determine $\lim_{x \rightarrow +\infty} f(x)$. Deduce an equation of an asymptote to (C).
- 2) Prove that $f'(x) = (2x - 1)e^{-x}$ and set up the table of variations of f.
- 3) Draw (C) and its asymptote.
- 4) The line (d) with equation $y = \frac{x}{2}$ intersects the curve (C) at one point only with abscissa α .

Verify that $3.5 < \alpha < 3.6$.

- 5) Let F be the function defined on $[0; +\infty[$ as $F(x) = (2x + 3)e^{-x} + 2x$.
 - a) Show that F is a primitive of f.
 - b) Deduce the area of the region limited by (C), the x-axis and the lines of equations $x = 0$ and $x = 1$.

Part B

In what follows, suppose that $\alpha = 3.55$

A factory produces souvenirs. The total cost of production of souvenirs C_T , expressed in millions LL, is modeled as $C_T(x) = f(x)$ **only** for all x in $[0.5 ; 4]$, where x , expressed in thousands, represents the number of souvenirs produced. ($0.5 \leq x \leq 4$).

1) Calculate, in LL, the total cost of production of 3 000 souvenirs. In this case, deduce the average cost of production of one souvenir.

2) The adjacent table is the table of variations of the profit function P of this factory, in millions LL, over $[0.5 ; 4]$.

a) Study if this factory can achieve a profit equal to 3 000 000 LL.

b) Determine the minimum number of souvenirs to be sold so that the factory realizes a profit.

c) The profit function P is defined over $[0.5 ; 4]$ as

$$P(x) = x - 2 + (2x + 1)e^{-x}.$$

Prove that the revenue function R , in millions LL, is modeled as $R(x) = x$ knowing that the whole production is sold.

d) Calculate the number of souvenirs that should be sold so that the revenue is double the total cost.

x	0.5	0.844	4
$P'(x)$		+	
$P(x)$	-1	0	2.164

المادة: رياضيات – لغة إنكليزية الشهادة: الثانوية العامة الفرع: الاقتصاد والاجتماع نموذج رقم: 1 / 2019 المدة: ساعتان	الهيئة الأكاديمية المشتركة قسم: الرياضيات	
---	--	--

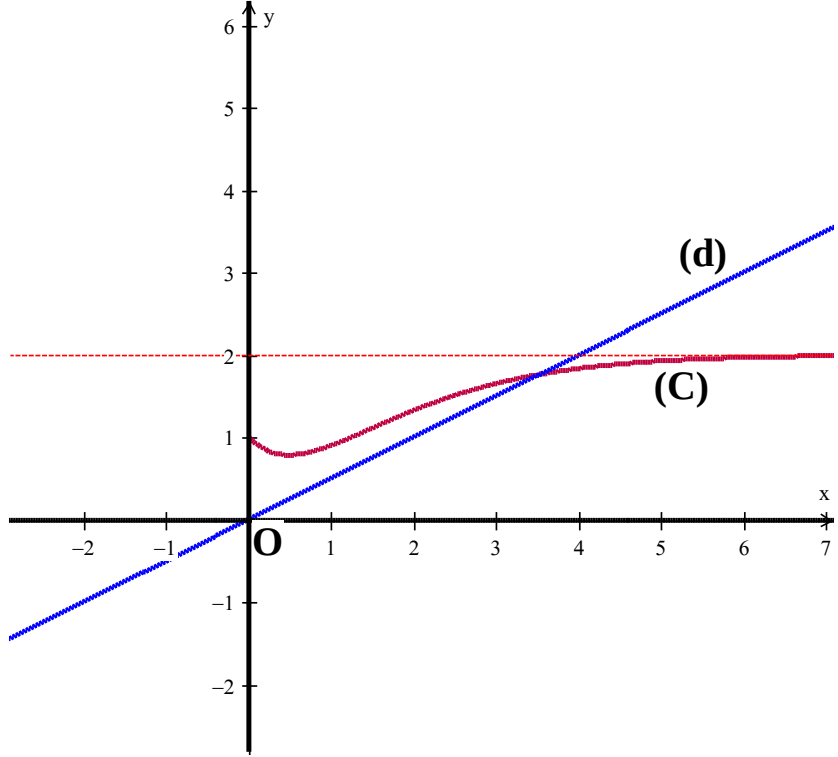
أسس التصحيح من 35

QI	Answers	Mark
1	$(D_{y/x}): y = 0.679x + 4.330$	0.5
2	Supply = $0.679(15) + 4.330 = 14, 515$ thousand articles, then 14515 articles	1.5
3	The equilibrium price is 3158 LL and the equilibrium quantity is 6474 articles.	0.75 0.75
4	Revenue = $(7)(-0.567(7) + 8.266)(1000) = 30079$ LL	1.5
5a	$e(x) = -x \cdot \frac{z'}{z} = \frac{567x}{-567x + 8266}$	1
5b	$e(4) = 0.378$. For a price of 4000 LL, if the price increases by 1% then the demand decreases by 0.378%.	0.25 0.75

QII	Answers	Mark
A1	$V_{n+1} = (1+0.5/100)V_n + 200000/1000000 = 1.005V_n + 0.2$	0.5
A2a	$W_{n+1} = 1.005W_n$; so $\alpha = 40$	1
A2b	$W_n = 44(1.005)^n$; and $V_n = 44(1.005)^n - 40$	0.25 0.25
A3a	$V_{10} = 6.250165$. So: 6250165 LL	1
A3b	$I = 6250165 - 4000000 = 2250165$ LL	0.5
A4	$V_{n+1} - V_n = 44(1.005)^{n+1} - 44(1.005)^n = 44(1.005)^n(1.005-1)$ $= 0.22(1.005)^n > 0$. Therefore (V_n) is an increasing sequence.	1.5
B1	The common ratio is 1.005 ; $U_n = U_0 \times q^n = 8(1.005)^n$	0.25 0.25
B2	$V_n > U_n$ gives $n > 21.12$ so $n = 22$. Thus after 22 years.	1.5

QIII	Answers	Mark
A1	$P(WWW \text{ or } RRR \text{ or } GGG) = \frac{A_4^3 + A_5^3 + A_3^3}{A_{12}^3} = \frac{3}{44}$	1
A2	(2G1R or 1G2R or 1G1R1W) where 2G1R can be written $\frac{3!}{2!} = 3$ ways: GGR, GRG, RGG and similarly for 2R1G. where RGW can be written in 3! ways $P = \frac{A_3^2 \times A_5^1 \times 3 + A_3^1 \times A_5^2 \times 3 + A_3^1 \times A_5^1 \times A_4^1 \times 3!}{A_{12}^3} = \frac{21}{44}$	1.5
B1	5000 – 10000 = -5000; the first 2 selected balls are not green and the game ends. 8000 + 2000 – 10000 = 0; the first 2 selected balls are green and the second 2 selected balls are of different color. 8000 + 12000 – 10000 = 10000. The first 2 selected balls are green and the second 2 selected balls are of the same color.	1
B2	$P(X = 0) = \frac{C_3^2}{C_{12}^2} \times \left(1 - \frac{C_4^2 + C_5^2}{C_{10}^2}\right) = \frac{29}{990}$. The first 2 balls are both green $\frac{C_3^2}{C_{12}^2}$ and the second 2 balls from 10 are of different color (RG, RW, WG) $\frac{C_5^1 C_1^1 + C_5^1 C_4^1 + C_4^1 C_1^1}{C_{10}^2} =$ $\left(1 - \frac{C_4^2 + C_5^2}{C_{10}^2}\right)$ “opposite of 2 balls same color”	1
B3	$P(X = -5000) = 1 - \frac{C_3^2}{C_{12}^2} = \frac{21}{22}$ $P(X = 10000) = \frac{C_3^2}{C_{12}^2} \times \frac{C_4^2 + C_5^2}{C_{10}^2} = \frac{8}{495}$ and $P(X = 0) = \frac{29}{990}$	2
B4	$EX = 0 + (-5000)(21/22) + (10000)(8/495) = -4611.11 < 0$ The player is therefore expected to lose.	0.5

QIV	Answers	Mark												
A1	$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} (-2xe^{-x} - e^{-x} + 2) = 2 ; y = 2 \text{ HA}$	1												
A2	$f'(x) = (-2 + 2x + 1)e^{-x} = (2x - 1)e^{-x}$ <table><tr><td>x</td><td>0</td><td>1/2</td><td>$+\infty$</td></tr><tr><td>f'(x)</td><td>-</td><td>0</td><td>+</td></tr></table> <table><tr><td>f(x)</td><td>1</td><td></td><td>$+\infty$</td></tr></table> 0.786	x	0	1/2	$+\infty$	f'(x)	-	0	+	f(x)	1		$+\infty$	2
x	0	1/2	$+\infty$											
f'(x)	-	0	+											
f(x)	1		$+\infty$											

A3		1.5
A4	$f(3.5) = 1.758 > 3.5/2$ $f(3.6) = 1.775 < 3.5/2$ Then $3.5 < \alpha < 3.6$	1
A5a	$F'(x) = (2 - 2x - 3)e^{-x} + 2 = (-2x - 1)e^{-x} + 2 = f(x)$	1
A5b	$\text{Area} = \int_0^1 f(x)dx = [F(x)]_0^1 = 5e^{-1} - 1 = 0.839 \text{ u}^2$	1.5
B1	$C_T(3) = 1.651490 \text{ LL}$; therefore 1 651 490 LL Average cost of production of a souvenir = $1651490/3000 \approx 550.5 \text{ LL}$	2
B2a	According to the table of variations of the profit function: $2164000 < 3000000$ therefore NO.	0.5
B2b	$P(0.844) = 0$ and P strictly increasing. For the production of 845 souvenirs the factory realizes profit.	1.5
B2c	$R(x) = P(x) + C(x) = x$	0.5
B2d	$R(x) = 2C(x)$ gives $f(x) = x/2$, then $x = \alpha$, therefore 3 550 articles.	1.5