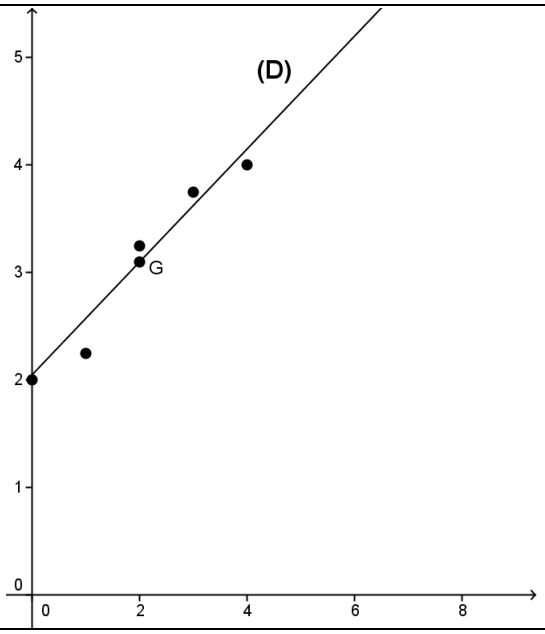
مشروع معيار التصحيح

مسابقة في مادة الرياضيات
المدة: ساعتان

I- (7 points)

Q	Correction	Grade
1-a	$\bar{x}=2\ ;\ \bar{y}=3.1$	**1**
1-b	$(D): y=0.525x+2.05$	**1**
2	$G(2,3,1)$ 	**2**
3	age of $31 \rightarrow x=6$ then $y=0.525(6)+2.05=5.2$ therefore 520 000 LL	**1.5**
4	$\frac{5.2-2}{2}=1.6$ then 160% is the increasing percentage	**1.5**

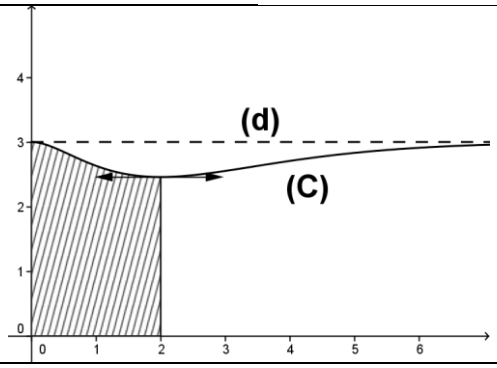
II- (7 points)

Q	Correction	Grade
1	$P(M)=1-P(W)=1-0.65=0.35$	**1**
2	$P(I\cap W)=P\left(\frac{W}{I}\right)\times P(I)=0.7\times 0.8=0.56$; $P(A\cap W)=P\left(\frac{W}{A}\right)\times P(A)=0.8\times 0.1=0.08$; $P(W\cap T)=P(W)-P(W\cap I)-P(W\cap A)=0.65-0.56-0.08=0.01$ $P\left(\frac{W}{T}\right)=\frac{P(W\cap T)}{P(T)}=\frac{0.01}{0.1}=0.1.$	**3**
3	$P\left(\frac{I}{M}\right)=\frac{P(I\cap M)}{P(M)}=\frac{0.8\times 0.3}{0.35}=0.68.$	**1.5**
4	$P(B)=\frac{C_{160}^2\times C_{20}^1+C_{160}^1\times C_{20}^2}{C_{200}^3}=...=\frac{1424}{6567}\approx 0.216.$	**1.5**

III- (7 points)

Q	Correction	Grade
1	$U_1 = \left(1 + \frac{0,06}{12}\right) \times 200 = 219,1$ et $U_2 = \left(1 + \frac{0.06}{12}\right) \times 219.1 = 218.1955.$	**1**
2	$U_{n+1} = \left(1 + \frac{0.06}{12}\right) \times U_n - 2 = 1.005\,U_n - 2$	**1**
3-a	$V_{n+1} = U_{n+1} - 400 = 1.005\,U_n - 402 = 1.005\left(U_n - 400\right) = 1.005\,V_n$ $q = 1.005$ and $V_0 = U_0 - 400 = 220 - 400 = -180$	**1.5**
3-b	$V_n = V_0 \times q^n = -180 \times (1.005)^n$ and $U_n = V_n + 400 = -180 \times (1.005)^n + 400$	**1**
3-c	$U_{n+1} - U_n = -180 \times (1.005)^{n+1} + 400 + 180 \times (1.005)^n - 400$ $= -180 \times (1.005)^n (1.005 - 1) = -0.9 \times (1.005)^n < 0$	**1**
3-d	$U_n < \frac{U_0}{2}$; $-180 \times (1.005)^n + 400 < 110$; $(1.005)^n > \frac{29}{18}$; $n > \frac{\ln(29/18)}{\ln(1.005)}$; $n > 95.6$; $n = 96$ then after 8 years.	**1.5**

IV- (14 points)

Q	Correction	Grade
A-1	$\lim\limits_{x \to +\infty} f(x) = 3$ then (d) : y = 3 is a horizontal asymptote at $+\infty$	**1**
A-2	$f'(x) = x(x-2)e^{-x}$ same sign as $x - 2$ then: x: 0 — 2 — $+\infty$ $f'(x)$: 0 — 0 + $f(x)$: 3 ↘ $3 - \frac{4}{e^2}$ ↗ 3	**1.5**
A-3		**1.5**
A-4-a	$F'(x) = (2x+2)e^{-x} - e^{-x}(x^2 + 2x + 2) + 3 = f(x)$	**1.5**
A-4-b	$A = \int_0^2 f(x)\,dx = \left[F(x)\right]_0^2 = F(2) - F(0) = \frac{10}{e^2} + 6 - 2 = \left(4 + \frac{10}{e^2}\right) u^2$	**1.5**
B-1	$f(6) = -36e^{-6} + 3 = 2.910765$ the average cost is: 2 910 765 LL.	**1.5**
B-2-a	According to the table of variations the average cost production is minimal for $x = 2$ then 200 liters. The minimal average cost is $f(2) = 2.45866$ then 245 866 LL	**1.5**
B-2-b	y = 2,3 < f(2) (The minimum of f) then no gain.	**1.5**
B-3-a	$C_T(x) = x\,f(x) = -x^3e^{-x} + 3x.$	**1**
B-3-b	$C_m(x) = \left(x^3 - 3x^2\right)e^{-x} + 3$; $C_M(x) = C_m(x)$; $\left(x^3 - 3x^2\right)e^{-x} + 3 = -x^2e^{-x} + 3$; $x^3 - 3x^2 = -x^2$; $x^2(x-2) = 0$; x = 0 or x = 2 but $0 \notin [0.2\ ;\ 9]$ then $x = 2$ therefore for a production of 200 liters.	**1.5**