

مسابقة في : تايلور ايرل ا قدام
الاسم: المدة : ساعتان
الرقم:

عدد المسائل : سبعة

ملاحظة : يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (1 point)

$$\text{Given that : } A = \frac{1 - \frac{2}{3}}{2 + \frac{1}{3}}, \quad B = \frac{5 \times 10^8 \times 2 \times 10^3}{7 \times (10^4)^3}.$$

Calculate A and B , showing all the steps of calculation, and deduce that A and B are two forms of the same number.

II-(1 ½ points)

$$\text{Given that : } C = \frac{\sqrt{45} - \sqrt{80} + 2\sqrt{125}}{\sqrt{7} \times \sqrt{35} - 7\sqrt{5} + 3}.$$

Calculate C and give the result in the form $a\sqrt{5}$ where a is an integer.

III-(1 point)

$$\text{Let } E = \frac{4x + 7}{3}.$$

1) Calculate the value of E for $x = \frac{7}{4}$.

2) Without solving the inequality $\frac{4x + 7}{3} < 5$, is the number $\frac{7}{4}$ a solution of this inequality?

Justify your answer.

IV- (2 points)

There are some missing numbers in the following text :

« For buying..... pencils and 2 pens we pay L L , and for buying pencils and 3 pens we pay 7800 L L. »

Setting up the complete given of the text in equations give the following system :

$$\begin{cases} 4x + 2y = 5600 \\ 2x + 3y = 7800 \end{cases}.$$

1) Copy again the complete text according to the given system.

2) Solve, showing all the steps of calculation, the preceding system and find the price of one pencil and the price of one pen.

V- (4 points)

In the plane of an orthonormal system of axes $x' O x$, $y' O y$, consider the points :

$A(-4 ; 4)$, $B(3 ; 3)$ and $C(1 ; -1)$.

1) Plot the points A, B and C.

2) Prove that the three points A, O and C are collinear.

3) Prove that the triangle ABC is isosceles of principal vertex A.

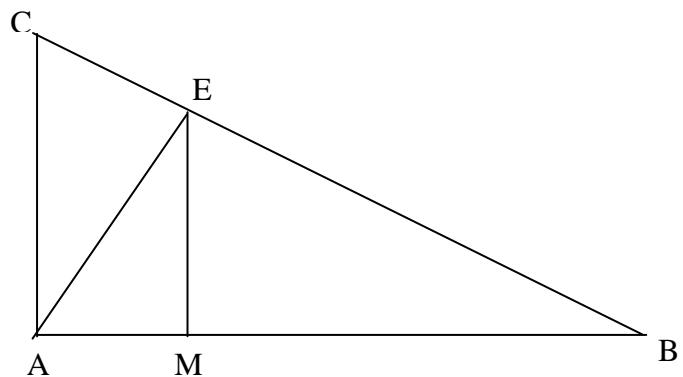
4) Let H be the midpoint of [BC] . Prove that (AH) is perpendicular to (BC).

5) Let N be the image (translate) of B by the translation of vector \overrightarrow{AC} . Prove that CABN is a rhombus.

VI- (5 ½ points)

In the opposite figure where the unit of length is the centimeter :

- ABC is a triangle right angled at A
- AB = 8 and AC = 4
- M is a point of [AB] such that $BM = x$ and $0 \leq x \leq 8$
- (ME) is perpendicular to (AB).



Part A

- 1) Prove that: $ME = \frac{1}{2}x$.
- 2) Calculate x so that the triangle AME is isosceles.

Part B

Consider an orthonormal system of axes $x' O x$, $y' O y$.

- 1) Draw, in this system, the straight line (d) of equation $y = \frac{x}{2}$ and the straight line (d') of equation $y = -x + 8$.
- 2) Using the graph, find again the result of question 2) of part A.

Part C

- 1) Calculate the exact value of the length of side [BC] of triangle ABC.
- 2) Write the value of BC appearing on your calculator, then give this value rounded to the nearest 10^{-2} by default.
- 3) Calculate $\tan \widehat{ABC}$, then calculate, rounded to the nearest degree, the angle \widehat{ABC} .

VII- (5 points)

EBF is a triangle right angled at B such that $EB = 6 \text{ cm}$, $BF = 8 \text{ cm}$ and $FE = 10 \text{ cm}$. M is the midpoint of [BF] and (\mathcal{C}) is the circle of diameter [MF]. The circle (\mathcal{C}) cuts again [EF] in G. The straight lines (MG) and (EB) intersect in S.

- 1) Draw a figure.
- 2) Prove that the four points E, B, M and G belong to the same circle whose diameter is to be determined .
- 3) a- Prove that the two triangles EBF and MGF are similar and calculate MG and GF.
b- Calculate the area of triangle MGF.
c- Calculate the ratio of the areas of the two triangles EBF and MGF.
- 4) Let P be the point of intersection of (EM) with (SF).
a- Prove that (EP) is perpendicular to (SF).
b- Deduce that P is a point of circle (\mathcal{C}) .

Questions		Eléments de réponses	Notes
I-		$A = \frac{1}{7}$; $B = \frac{1}{7}$.	$\frac{1}{2} + \frac{1}{2}$
II-		$C = \frac{3\sqrt{5} - 4\sqrt{5} + 10\sqrt{5}}{7\sqrt{5} - 7\sqrt{5} + 3} = \frac{9\sqrt{5}}{3} = 3\sqrt{5}.$	$1 \frac{1}{2}$
III-	a)	$E = \frac{14}{3}$	$\frac{1}{2}$
	b)	$\frac{7}{4}$ sol de l'inéquation car $\frac{14}{3} < 5$ ou ...	$\frac{1}{2}$
IV	1)	Pour acheter 4 crayons et 2 stylos on paye 5600 L L, et pour acheter 2 crayons et 3 stylos on paye 7800 L L.	$\frac{1}{2}$
	2)	$4y = 10000$; $y = 2500$; $x = 150$ prix d'un crayon 150 L L Prix d'un stylo 2500 L L .	$1 \frac{1}{2}$
V-	1)	figure	$\frac{1}{2}$
	2)	équation (OA) : $y = -x$; C est un point de (OA) ou ...	1
	3)	$AB = \sqrt{50} = 5\sqrt{2}$; $AC = \sqrt{50} = 5\sqrt{2}$;	1
		$AB = AC$ donc ABC est un triangle isocèle en A.	
	4)	[AH] mediane à la fois hauteur .	$\frac{1}{2}$
	5)	$\overrightarrow{AC} = \overrightarrow{BN}$ (CABN parallélogramme). $AB = AC$ donc CABN est un losange ou	1
VI-	A- 1)	(ME) // (AC) ; $\frac{BM}{BA} = \frac{BE}{BC} = \frac{ME}{AC}$ (Thalés ou triangle semblables) $ME = \frac{x}{2}$.	$\frac{3}{4}$
	2)	$8 - x = \frac{x}{2}$; $x = \frac{16}{3}$.	$\frac{3}{4}$
	B- 1)	Tracé : (d) ; (d').	$1 \frac{1}{2}$
	2)	I est le point d'intersection de (d) et (d') ; $x_I = \frac{16}{3}$	$\frac{1}{2}$
	C- 1)	$BC^2 = 64 + 16 = 80$ $BC = 4\sqrt{5}$ cm	$\frac{1}{2}$
	2)	La valeur affichée est 8,94427191	
		La valeur approchée à 10^{-2} près par défaut est 8,94 cm	$\frac{1}{2}$

Questions		Eléments de réponses	Notes
Suite VI	C- 3)	Tan \widehat{ABC} = $\frac{AC}{BA} = \frac{4}{8} = 0,5$; $ABC = \tan^{-1}(0,5) = 26,56$; $ABC \cong 27^\circ$	1
VII-	1)	Figure	$\frac{1}{2}$
	2)	MGE = MBE = 90° ; GME et MBE 2 triangles de même hypotenuse [EM] B ; M ; G et E sur le même cercle de Diamètre [ME].	$\frac{3}{4}$
	3) a-	$B = G = 90^\circ$; F commun ; $\frac{GF}{BF} = \frac{FM}{FE} = \frac{GM}{BE} = \frac{2}{5}$ $GF = \frac{16}{5}$; $GM = \frac{12}{5}$	$1\frac{1}{4}$
	3) b-	$A(GMF) = \frac{GF \times GM}{2} = \frac{96}{25} = 3,84 \text{ cm}^2$	$\frac{1}{2}$
	3) c-	$\frac{A(GMF)}{A(EBF)} = \frac{GF \times GM}{BF \times BE} = \frac{4}{25} = \left(\frac{2}{5}\right)^2$ ou ...	$\frac{3}{4}$
	4) a-	[SG] et [BF] sont 2 hauteurs du triangle ESF alors [EP] est la 3ème hauteur.	$\frac{3}{4}$
	4) b-	MPF = 90° [MF] diamètre de (\mathcal{C}) donc P est un point de (\mathcal{C}).	$\frac{1}{2}$