

المادة: الرياضيات الشهادة: المتوسطة نموذج رقم -6- المدة : ساعتان	الهيئة الأكاديمية المشتركة قسم : الرياضيات	 المركز التربوي للبحوث والإنماء
---	---	---

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (2 points)

Answer "true" or "false" and justify your answer.

- 1) The solution of the inequality $\frac{-2x+3}{-3} \leq \frac{x+1}{-3}$ is $x \geq \frac{2}{3}$.
- 2) The price of an object becomes 90000 LL after two successive reductions of 20%. Its initial price is 150000 LL.
- 3) If $x^2 = \frac{\sqrt{15}}{\sqrt{14}} + \frac{5}{7} \left(1 - \frac{3}{10}\right)^2$, then $x = \frac{3\sqrt{15}}{10}$ or $x = -\frac{3\sqrt{15}}{10}$.
- 4) $\left(-\sqrt{\frac{5}{2}} - x\sqrt{\frac{1}{2}}\right)^2 = \frac{1}{2}(\sqrt{5} - x)^2$.

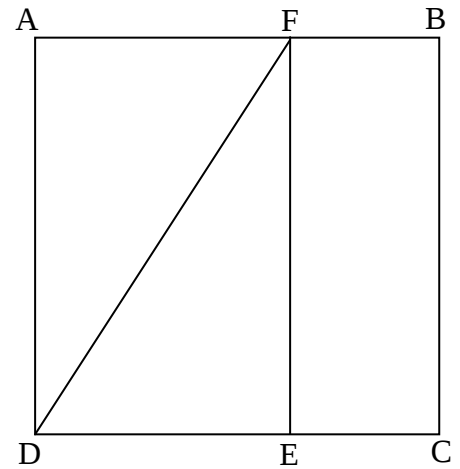
II- (3points)

Let x be a number that is greater than or equal to 4.

ABCD is a square.

AFED is a rectangle, where $DF^2 = 5x^2 - 10x + 10$.

- 1) If $AF = x+1$, show that the side of the square ABCD is $2x - 3$.
- 2) Prove that F cannot be the midpoint of [AB].
- 3)
 - a) Show that the area A of the rectangle BCEF is expressed by the relation:
 $A = (2x - 3)^2 - (2x - 3)(x + 1)$.
 - b) Factorize A.
 - c) For which value of x does the area of the rectangle BCEF become one third of that of the triangle AFD?



III- (3points)

The director of a school organizes a trip for grade 9 students at the end of the year. He decides not to make the trip if the percentage of participants is less than 70% of all grade nine students.

The table below shows the answers of each section.

Section	Total Number of Students	Answer
Gr. 9A	35 students (among them 20 girls)	$\frac{2}{5}$ of the girls and $\frac{1}{5}$ of the boys will not participate.
Gr. 9B	24 students (among them 14 boys)	50% of the girls and $\frac{2}{7}$ of the boys will not participate.
Gr. 9C	30 students (among them 15 boys)	60% of the girls and 80% of the boys will participate.

- 1) In each section, find the number of students who will participate in this trip.
- 2) Will the director of the school make the trip?

IV- (2 points)

A bag contains x red balls and y blue balls.

If we replace 5 blue balls by 5 red balls, the number of red balls will be twice the number of blue balls.

If we take 3 red balls from the bag, the number of blue balls will be twice the number of red balls.

- 1) Choose the system that models the text given above .

$$\begin{cases} x + 5 = 2y \\ 2(x - 3) = y \end{cases} \text{ or } \begin{cases} x + 5 = 2(y - 5) \\ 2(x - 3) = y \end{cases}$$

- 2) Calculate x and y .

V- (5 points)

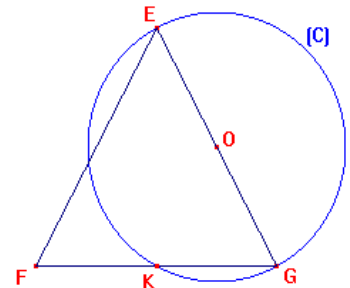
In an orthonormal system of axes $x'Ox$ and $y'Oy$, consider the line (D) with equation $y = -2x + 4$ and the two points $I(1 ; 2)$ and $C(4 ; 4)$.

- 1) (D) intersects $x'Ox$ at A and $y'Oy$ at B . Calculate the coordinates of the two points A and B , then draw line (D) .
- 2) Verify that I is the midpoint of segment $[AB]$.
- 3)
 - a) Write an equation of the median issued from point O in triangle OAB .
 - b) Calculate, to the nearest one degree, the measure of the angle that line (OI) makes with the axis $x'Ox$.
- 4) Let (D') be the perpendicular bisector of segment $[BC]$ that intersects it at J .
 - a) Write an equation of (D') .
 - b) Deduce that $AB = AC$.
- 5) Let L be the orthogonal projection of point I on the axis $x'Ox$. Show that the two triangles ILA and AJC are similar. Deduce that $AC = 2OI$.

VI- (5 points)

In the next figure, EFG is an isosceles triangle with vertex E , where $FG = 5\text{cm}$ and $EG = 6\text{cm}$.

The circle (C) with center O and diameter $[EG]$ intersects the segment $[FG]$ at K .



- 1) Reproduce the figure in real measures.
- 2)
 - a) Show that K is the midpoint of segment $[FG]$.
 - b) Calculate the value of EK to the nearest millimeter.
- 3) Let S be the image of K under the translation of vector $\overrightarrow{FE}$.
 - a) Plot the point S on the figure.
 - b) Prove that $ESGK$ is a rectangle.
- 4) Let P be a point on segment $[EG]$ distinct from O . The parallel through P to (FG) intersects (EF) at R . Suppose that x is the length, expressed in cm, of segment $[EP]$.
 - a) What is the nature of triangle EPR ? Justify your answer.
 - b) Prove that $PR = \frac{5x}{6}$ and express, in terms of x , the perimeter of triangle EPR .
 - c) Show that the perimeter of the trapezoid $RPGF$ is equal to $\frac{-7x}{6} + 17$.
 - d) Can you find a position for point P on segment $[EG]$ so that the triangle and the trapezoid have the same perimeter? Justify your answer.

المادة: الرياضيات الشهادة: المتوسطة نموذج رقم -6- المدة : ساعتان	الهيئة الأكاديمية المشتركة قسم : الرياضيات	 المركز التربوي للبحوث والإنماء
---	---	--

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

Question I		Mark
1	False because when we multiply by a negative number, the inequality changes.	0.5
2	False because $\frac{90000}{0.64} = 140625$ LL .	0.5
3	True because $x^2 = 1 + \frac{5}{7} \times \frac{7^2}{10^2}$, then $x = \frac{3\sqrt{15}}{10}$ or $-\frac{3\sqrt{15}}{10}$.	0.5
4	Faux because it is $\frac{5}{2} + \frac{x^2}{2} + \frac{2x\sqrt{5}}{2} = \frac{1}{2}(\sqrt{5} + x)^2$.	0.5

Question II		Mark
1	Using Pythagoras: $AD^2 = DF^2 - AF^2$, alors $AD = 2x - 3$	0.5
2	$AB=2AF$, then $2x - 3=2x+ 2$ has no solution	0.5
3.a	$A_{BCEF} = A_{ABCD} - A_{AFED} = (2x - 3)^2 - (2x - 3)(x + 1)$	0.5
3.b	$A=(2x - 3)(x - 4)$	0.5
3.c	$(2x-3)(x-4) = \frac{(x+1)(2x-3)}{6}$; $x = 5$.	1

Question III		Mark
1	In Gr. 9A: The number of students who will participate is $\frac{3}{5} \times 20 + \frac{4}{5} \times 15 = 24$	0.5
	In Gr. 9B: The number of students who will participate is $5 + \frac{5}{7} \times 14 = 15$	0.5
	In Gr. 9C: The number of students who will participate is $\frac{3}{5} \times 15 + \frac{4}{5} \times 15 = 21$	0.5
2	The number of students who will participate in Gr. 9 is 60 The percentage is $\frac{60}{89} \times 100 = 67.41\%$ and the director will not make the trip.	0.5 1

Question IV		Mark
1	$\begin{cases} x + 5 = 2(y - 5) \\ 2(x - 3) = y \end{cases}$	1
2	$x=9$ and $y=12$	1

Question V		Mark
1	A(2 ;0) and B(0 ; 4) (D) passes through A and B	0.75
2	$\frac{x_A + x_B}{2} = 1 = x_I$ and $\frac{y_A + y_B}{2} = 2 = y_I$	0.5
3.a	(OI): $y=2x$	0.5
3.b	$\tan \alpha = 2 = a_{(OI)}$, then $\alpha = \tan^{-1} 2 = 63.43 \approx 63^\circ$	0.5

4.a	$y_C = y_B = 4$, then (BC): $y = 4$, $x_J = \frac{x_C + x_B}{2} = 2$. Thus (D'): $x = 2$	0.75
4.b	$x_A = 2$, then (D') passes through A. Then $AB = AC$.	0.5
5	ILA and AJC are similar because $\hat{L} = \hat{J} = 90^\circ$ $\hat{C} = \hat{A}$ since the triangle ABC is isosceles ($\hat{C} = \hat{B}$ and $\hat{B} = \hat{A}$ (alternate interior)) Ratio of similitude: $\frac{IL}{AJ} = \frac{AL}{JC} = \frac{IA}{AC} = \frac{1}{2}$, then $AC = 2AI$ but $AI = OI$.	1 0.5

Question VI		Mark
1		0.5
2.a	EKG is a right triangle at K (inscribed in a semi-circle) and EFG is an isosceles triangle with vertex E, then [EK] is an altitude and median, then K is the midpoint of [FG]	0.5
2.b	By Pythagoras: $EK^2 = EG^2 - KG^2$, then $EK = 5.45$	0.5
3.a	$\overrightarrow{KS} = \overrightarrow{FE}$ (by translation)	0.5
3.b	$\overrightarrow{ES} = \overrightarrow{FK}$ since ESFK is a parallelogram, and $\overrightarrow{KG} = \overrightarrow{FK}$ since K is the midpoint of [FG], then $\overrightarrow{ES} = \overrightarrow{KG}$ and $\hat{K} = 90^\circ$. Thus ESGK is a rectangle.	0.75
4.a	EPR is an isosceles triangle since $\hat{R} = \hat{F}$ and $\hat{P} = \hat{G}$ (corresponding angles), but $\hat{G} = \hat{F}$, thus $\hat{R} = \hat{P}$.	0.5
4.b	By Thales: $\frac{EP}{EG} = \frac{ER}{EF} = \frac{RP}{FG} = \frac{x}{6}$, then $RP = \frac{5x}{6}$.	0.75
4.c	$\frac{5x}{6} + 6 - x + 6 - x + 5 = \frac{-7x}{6} + 17$	0.5
4.d	$17 = \frac{-7x}{6} + 17$, then $x = 0$. Thus a position for P cannot be found.	0.5