المادة:الرياضيات الشهادة: المتوسطة نموذج رقم -3- المدّة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (3 points)

Answer « true » or « false » andjustify your answer.

1) $(-2x-2)^2 = 4\,(x+1)^2$.
2) The solutions of the equation $x^2+10=0$ are $\sqrt{10}$ and $-\sqrt{10}$.
3) Ifx is an acute angle and $\sin x = \frac{1}{3}$, then$\cos x = \frac{2}{3}$.
4) The equation $(x+3)^2 = 0$has no solution.
5) If $x$is a number greater than3, then$(x^2+1)(2x-5)$is positive.

II- (2 points)

The questions 1) et 2) are independent.Show all the steps of your work.

1)Given$A = \frac{1}{\sqrt{7}+1} + \frac{1}{\sqrt{7}-1}$and B$= \frac{7}{3\sqrt{7}}$.

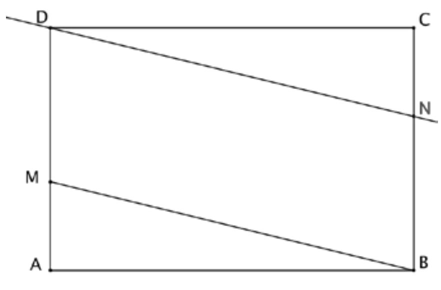
Compare A and B.

2) a) Verify that: $\frac{4\sqrt{2}+2}{4+\sqrt{2}} = \sqrt{2}$.

b)Use the previous equality to prove that$\frac{(\sqrt{32}+2)^2}{(\sqrt{36-10-\sqrt{2}})^2}$is a natural number.

III- (4 points)

ABCD is a rectangle such that AB = 4 m and AD = 3 m.
M is a point on [AD]. The parallel through D to (BM) intersects [BC] at N.Let AM = x.



Part A

1) Prove that:

a) $x$isless than 3.

b) DMBN is a parallelogram.

c) NC = x.

2) Provethat the area ofthe square with side DM is $(3-x)^2$.

3) Prove that the area of the parallelogram DMBN is equal to$12-4x$.

Part B

1)Factorize S'– S.

2)Can you find x so that the two areas are equal?

3)a) Solve the equation (x+1)(3-x) = 3 .

b)Give a geometric interpretation to the result.

IV- (2 points)

The sum of two numbers is 47. When we divide one of the numbers by 2 and the other by 3, the sum becomes 17.5.

1) Which one of these 3 systems is related to the given?

$$\begin{cases} x+y=47 \\ 3x+2y=17{,}5 \end{cases} \qquad \begin{cases} y=47-x \\ 3x+2y=105 \end{cases} \qquad \begin{cases} x+y=47 \\ \dfrac{x+y}{2}+\dfrac{y}{3}=17{,}5 \end{cases}$$

2)Find the two numbers.

V- (4 points)

In an orthonormal system of axes $x'Ox$ and $y'Oy$), considerthe line (D) withequation $y=2x-1$, and the points B(2 ;3) and C(3 ;1) .

1)Draw the line (D) and plot the points B and C.

2)Does the line (D) pass through the points B and C? Justify.

3)Let (D') be the line with equation y= $-\frac{1}{2}x+\frac{5}{2}$.

a)Prove that (D') passesthrough C and is perpendicularto (D) .

b) (D') and (D) intersectat S. Determinethe coordinates of the point S .

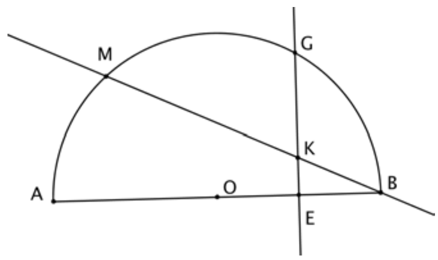
4) Determinethe coordinates of the point I, centreof the circlecircumscribedabout the triangle BSC, and determine the length of its radius.

5) Determine the coordinates of point A such that BSCA is a parallelogram. Prove that A is a point on the circlecircumscribedabout BSC.

VI- (5 points)

In the adjacent figure :

- a semicircle with diameter[AB]andcenter O;
- AB = 2R;
- E is midpoint of [OB];
- (GE) perpendicular bisector of [OB] (G is a point on the semicircle);
- K is a point onsegment [EG]. The line (BK) and the semicircleintersect at M.



1) Draw a figure,to be completed in the remaining parts of the problem.

2)a) Prove that the triangle OBG is equilateral.

b)Calculate GE in terms of R.

c)Calculate the angle GMB.

3)Prove that thetriangles BEK and BMA are similar.

Deduce that BK x BM = R^2.

4) The perpendicular through E to (AM) intersects (AM) at N.

Calculate the ratio $\dfrac{EN}{AM}$

5) In this part, suppose that k is the centroid of the triangle GOB.

a)Calculate EN and MN in terms of R.

b) Find R so that the perimeter of the quadrilateral AMNE is equal to $7\sqrt{3}+3$

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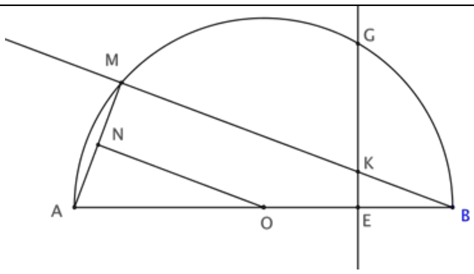
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

Answer keys

I.	**1)**	True Expand both expressions or: $(-2x-2)^2 = [-2(x+1)]^2 = 4\,(x+1)^2$	**0,5**
	2)	False The equation has no solution : a square can't be negative .	**0,5**
	3)	False $(cosx)^2 = 1 - (sinx)^2 = \frac{8}{9}$; $cosx = \frac{2\sqrt{2}}{3}$	**0,75**
	4)	False only-3 is the solution.	**0,5**
	5)	True because : $x^2 + 1 > 0$for all$x$;and $2x - 5 > 0$and$x$ > 2,5. The product of two positive numbers is positive.	**0,75**
II.	**1)**	$A = \frac{2\sqrt{7}}{6} = \frac{\sqrt{7}}{3} = \frac{\sqrt{7}\times\sqrt{7}}{3\sqrt{7}} = \frac{7}{3\sqrt{7}}$, hence $A = B$	**0,75**
	2-a	we can show that: $\sqrt{2} \times (4 + \sqrt{2}) = 4\sqrt{2} + 2$	**0,5**
	2-b	$\frac{(\sqrt{32}+2)^2}{(\sqrt{36}-10-\sqrt{2})^2} = \frac{(4\sqrt{2}+2)^2}{(6-10-\sqrt{2})^2} = \frac{(4\sqrt{2}+2)^2}{(-4-\sqrt{2})^2} = \frac{(4\sqrt{2}+2)^2}{(4+\sqrt{2})^2} =$ $\left(\frac{4\sqrt{2}+2}{4+\sqrt{2}}\right)^2 = (\sqrt{2})^2 = 2$,and 2 is a natural number.	**0,75**
III.	**A.1-a**	AD = 3 and $x$is positive, hence$x$is between 0 and 3.	**0,25**
	A.1-b	DMBN is a parallelogramsince opposite sides are parallel.	**0,5**
	A.1-c	AD =BC , since ABCD is a rectangle ;and DM = NB, since DMBN parallelogram. Therefore AD –DM =BC – NB, and AM = NC = x	**0,5**
	2)	DM = 3-x , then area of the square = $(3-x)^2$.	**0,25**
	3)	Different ways: Area (DMBN) = Area (ABCD) – 2 x Area (AMB), becausethe triangles AMB and DCN are congruent. Area (DMBN) = 12 - 4 x	**0,5**
	B-1	$(12-4x)-(3-x)^2- = (3-x)(x+1)$.	**0.5**

	B-2	x=3 or x = -1, both are rejected.	**0,5**
	B-3-a	(x+1)(3-x) = 3 , then x=2.	**0,5**
	B-3-b	The area of parallelogram is 3 more than the area of square.	**0,5**
IV	**1)**	$\begin{cases} x+y=47 \\ \frac{x}{2}+\frac{y}{3}=17{,}5 \end{cases}$ same as $\begin{cases} y=47-x \\ 3x+2y=105 \end{cases}$	**1**
	2)	The two numbers are 7 and 40	**1**
V	**1)**	Figure.	**0,5 + 0,25**
	2)	For $x=2$, $2x-1=3$; hence B is on (D). For $x=3$, $2x-1=5$; hence C is not on (D).	**0,25 0,25**
	3) a	For $x=3$, $-\frac{1}{2}x+\frac{5}{2}=1$; hence C is on (D').	**0,25**
	3) b	$\begin{cases} y=2x-1 \\ y=-\frac{1}{2}x+\frac{5}{2} \end{cases}$ therefore $2x-1=-\frac{1}{2}x+\frac{5}{2}$; $x=\frac{7}{5}$ et $y=\frac{9}{5}$ and S($\frac{7}{5}$; $\frac{9}{5}$)	**0,25 0,5**
	4)	I midpoint of [BC]. Therefore I($\frac{5}{2}$; 2) R = IB = $\frac{\sqrt{5}}{2}$	**0,25 0,5**
	5)	$\overrightarrow{BA}=\overrightarrow{SC}$, therefore : $x-2=3-\frac{7}{5}$ and $y-3=1-\frac{9}{5}$ $x=\frac{18}{5}$ and $y=\frac{11}{5}$ BSCA rectangle, so BSC is a right triangleat A ; and A is on the circle circumscribed about triangle BSC	**0,5** **0,5**
VI	**1)**		**0,5**
	2)a-	OG = BG (perpendicular bisector) ; OG = OB (radii) Therefore OG = BG = OB, and OBG equilateral. 2)b- $GE=\frac{R\sqrt{3}}{2}$. 2)c-GMB = $\frac{GOB}{2}$ = 30 degree.	**0,75**
	3)	BMA and BEK areright triangles and they have a common angle $\hat{B}$. $\frac{BA}{BK}=\frac{AM}{KE}=\frac{MB}{EB}$;	**0,75** **0,5 + 0,5**

		therefore $BK \times BM = BA \times EB = 2R \times \frac{R}{2} = R^2$	
	4)	$\frac{EN}{AM} = \frac{BE}{BA} = \frac{1}{4}$.	**0,75**
	5-a	AM= $\frac{AB}{2} = R$ and EN = $\frac{R}{4}$. BN = $\frac{1}{4}BM = \frac{R\sqrt{3}}{4}$ and MN = $\frac{3R\sqrt{3}}{4}$.	**0,5**
	5-b	R + $\frac{3R\sqrt{3}}{4} + \frac{R}{4} + \frac{3R}{2} = 7\sqrt{3} + 3$. R = $\frac{4(7\sqrt{3}+3)}{11+3\sqrt{3}}$.	**0,5 + 0,25**