المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة:الرياضيات الشهادة: المتوسطة نموذج رقم -2- المدّة : ساعتان

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (4 points)

Remark: The four parts of this question are independent.

1) Given: $A = 3^{24} + 3^{25} + 3^{26}$. Factorize A and deduce that A is a multiple of 13.
2) Given: $a = \sqrt{14} + \sqrt{2}$ and $b = -\sqrt{\frac{7}{2}} - \sqrt{\frac{1}{2}}$.
   - a) Calculate a^2 and b^2. Show your work.
   - b) Compare $\frac{a}{2}$ and b.
3) What is the sign of $\frac{-2x^2}{-y}$, when $x < 0$ *and* $y > 0$?
4) A Grade 9 student wrote: "If the price of one ball decreases by 15%, then the price of 2 balls will decrease by 30%." Is he right? Justify.

II- (4 points)

Part A

Let $A(x) = (1 - x)(6x - 8) + 16 - 9x^2$.

1) Expand and reduce $A(x)$.
2) Factorize $A(x)$.
3) Solve the following equations: A(x) = 0; A(x) = 8.

Part B

Given $E(x) = \frac{(x-3)(-2-5x)}{(2+5x)\ (4-3x)}$.

1) Determine the values of x for which E(x) is defined.
2) Simplify E(x).
3) Solve the equation: $E(x) = \frac{1}{3}$.

III- (4 points)

ABC is a right triangle right at A such that AC = 2 AB. Let AB = a, with $a > 0$.

[AM] is the median relative to [BC].

1) Draw a figure, to be completed by the remaining parts of the problem.
2) Calculate, in terms of a, BC and AM.
3) Draw (d), the perpendicular line to (AM) at A. Then, draw the perpendicular lines to (d) passing through the two points B and C; these lines intersect (d) in B' and C' respectively.
   - a) Prove that [BA) and [CA) are the respective bisectors of angles $\widehat{CBB'}$ and $\widehat{BCC'}$.
   - b) Prove that the triangles ACC' and ABB' are similar and find their ratio of similarity.
4) [AH] is the altitude in the triangle ABC.
   - a) Prove that $AH = \frac{2a\sqrt{5}}{5}$.
   - b) Calculate, in terms of a, the lengths $B'C'$, BB' and CC'.

IV- (4 points)

The planeis referredto an orthonormal system x′Ox, y′Oy.

1) Plot thethree pointsA(3 ; 2), B(2 ; 5),andC(−1 ; 4).
2) Prove that the triangle ABC is right isosceles.
3) Let Sbe the midpointof [AC], and Dbe the symmetricof B with respect to S.
   a) Determinethe nature ofthe quadrilateral ABCD.
   b) Calculate the coordinates of point D.
4) The parallel line to (AC) passing through D intersectsthe x-axisat a point T.
   a) Determine an equation of line (DT).
   b) Calculate the coordinates of point T.
5)
   a) Solve the equation: $x^2 = 5$.
   b) (1; m) are the coordinates of a point M, where m is a real number. Determinethe values of m so that M is on the circle circumscribed about the square ABCD.

V- (4 points)

Remark: The questions 1) and 2) are independent.

1) A team of 6 girlsand 4 boysdid the same test in mathematics. Themeanof the girls on this test is 14 and the mean of boys is 12.
   What is the mean of the team in this test?

2) Grade 7 students handle a survey on the number ofmobile phonesper family.

Number ofmobile phones	**0**	**1**	**2**	**3**
Numberof families	5	15	50	30

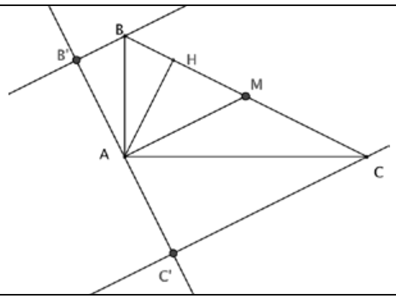
   a) What is the total frequency of the above distribution?
   b) What is the percent of families having at least 2 mobile phones?

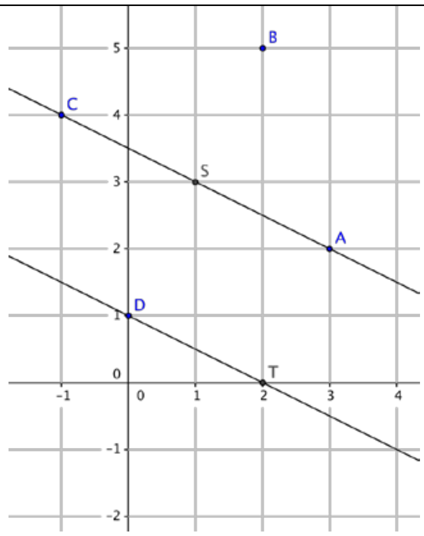
Representthis situation in a circle graph. Write down the calculationof the different Answer Key

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

		Key answers	Mark
I-	1)	$A = 3^{24}(1+3+9) = 3^{24} \times 13$ A is a multiple of 13 since 3^{24} is a naturel number.	0.5 0.5
	2) a)	$a^2 = 4(4+\sqrt{7})$; $b^2 = \frac{7}{2}+\frac{1}{2}+2\sqrt{\frac{7}{2}}\sqrt{\frac{1}{2}} = 4+\sqrt{7}$;$a^2 = 4b^2$	0.25 0.5
	2) b)	$a = -2b$	0.25
	3)	for all x ; $x^2 > 0$ *then* $-2x^2 < 0$ $-y < 0$, hence $\frac{-2x^2}{-y}>0$	0.5 0.25 0.25
	4)	False. Before reduction: x is the price of a ball and 2x the price of 2 balls. After reduction: 1 ball: 0,85 x ; Price of 2 balls: $(0,85\,x) \times 2 = 0,85 \times (2x)$ The price of 2 balls decreased by 15% We can think in this way: if this student was right and if he buys 7 balls, then the seller should pay him because the reduction will be 105%	1

		Part A	Mark
II-	1)	$A(x) = -15x^2 + 14x + 8$	0.25
	2)	$A(x) = (1-x)(6x-8) + 16 - 9x^2$ $A(x) = 2(1-x)(3x-4) + (4-3x)(4+3x)$ $A(x) = 2(1-x)(3x-4) - (3x-4)(4+3x)$ $A(x) = (3x-4)(-2-5x)$	0.25 0.25 0.25
	3)	$A(x) = 0$, for $x = \frac{4}{3}$ or $x = -\frac{2}{5}$ $A(x) = 8$ $-15x^2 + 14x + 8 = 8$ $-15x^2 + 14x = 0$ $x(-15x+14) = 0$ $x = \frac{14}{15}$ or $x = 0$	0.25 0.25 0.25 0.25 0.25 0.5
		Part B	
	1)	$x \neq \frac{4}{3}$ and $x \neq -\frac{2}{5}$	0.5
	2)	$E(x)= \frac{x-3}{3x-4}$	0.25
	3)	$\frac{x-3}{3x-4} = \frac{1}{3}$, then $3x - 9 = 3x - 4$; the equation has no solution	0.5

III-	1)			0.25
	2)		Pythagoras: AC = 2a, so $BC^2 = 4a^2 + a^2$; with BC > 0, therefore $BC = a\sqrt{5}$ $AM = \frac{a\sqrt{5}}{2}$	0.25 0.25
	3)	a)	BMA isosceles and M is the principal vertex, therefore $\widehat{MAB} = \widehat{MBA}$; (AM) //(BB'), $\widehat{B'BA}$ and $\widehat{MAB}$ are alternate-interior angles, thus $\widehat{B'BA} = \widehat{MAB}$ then $\widehat{B'BA} = \widehat{MBA}$ and [BA) is the bisector of $\widehat{CBB'}$. Same for: [CA) bisector of $\widehat{BCC'}$.	0.5
		b)	Right angle and $\widehat{B'BA} = \widehat{CAC'}$, same complements $\frac{BB'}{AC'} = \frac{B'A}{C'C} = \frac{AB}{CA} = \frac{a}{2a} = \frac{1}{2}$	0.5 0.25
	4)	a)	By areas of ABC: $\frac{1}{2}AH \times BC = \frac{1}{2}\ AB \times AC$ OR Using the similar triangles BAH and ABC. $AH \times BC = AB \times AC$ and $AH = \frac{AB \times AC}{BC} = \frac{2a^2}{a\sqrt{5}} = \frac{2a}{\sqrt{5}} = \frac{2a\sqrt{5}}{5}$	0.5 0.5
		b)	A is a point on the bisector of $\widehat{CBB'}$; therefore A is at the same distance from the sides of the angle. So $AH = AB'$. Same for $AH = AC'$. Therefore $B'C' = 2AH = \frac{4a\sqrt{5}}{5}$ $BB' = \frac{1}{2}\ AC'$, using 3) b) then $BB' = \frac{a\sqrt{5}}{5}$ $CC' = 2B'A$, using 3) b). Hence $CC' = 2 \times \frac{2a\sqrt{5}}{5}$, using 4) a) $CC' = \frac{4a\sqrt{5}}{5}$	0.25 0.25 0.25 0.25

IV	1)		 although it is not requested, we have completed the drawing.	0.25
	2)		$AB^2 = (2-3)^2 + (5-2)^2 = 1 + 9 = 10$; $AC^2 = 16 + 4 = 20$ $BC^2 = 9 + 1 = 10$; AB= BC et $AB^2 + BC^2 = AC^2$ Therefore ABC is right isosceles. The right angle is at B.	0.5 0.25
	3)	a)	ABCD is a square.	0.5
		b)	Different ways for calculation: using the midpoint S(1 ; 3) or $\overrightarrow{AD} = \overrightarrow{BC}$; D(0 ; 1)	0.5

	4)	**a)**	(DT) // (AC),slope: a $= \frac{y_A - y_C}{x_A - x_C} = \frac{-2}{4} = -\frac{1}{2}$ and D belongs to (DT), therefore $y = -\frac{1}{2}x + 1$.	**0.5** **0.25**
		b)	Ordinate of T is equal to zero. T(2 ;0)	**0.25**
	5)	**a)**	x =$\sqrt{5}$ or x=-$\sqrt{5}$	**0.25**
		b)	S(1 ;3) ; $SA^2 = 5$ $MS^2 = SA^2$; $MS^2 = 5$ $(1-1)^2 + (m-3)^2 = 5$; $(m-3)^2 = 5$; $m - 3 = \sqrt{5}$ or $m - 3 = -\sqrt{5}$; $m = 3 + \sqrt{5}$ or $m = 3 - \sqrt{5}$	**0.25** **0.25** **0.25**

<table>
<tr><td rowspan="4">V</td><td>1)</td><td colspan="2">[(6 × 14) + (4 × 12)] ÷ 10 = 13.2</td><td>1.5</td></tr>
<tr><td rowspan="3">2)</td><td>a)</td><td>5 + 15 + 50 + 30 = 100</td><td>0.25</td></tr>
<tr><td>b)</td><td>(50 + 30) ÷ 100 = 0.8 or 80%</td><td>0.5 +
0.25</td></tr>
<tr><td>c)</td><td>Calculation of angles:
<table>
<tr><td>Number of mobile phones</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Number of families</td><td>5</td><td>15</td><td>50</td><td>30</td></tr>
<tr><td>Angles</td><td>18°</td><td>54°</td><td>180°</td><td>108°</td></tr>
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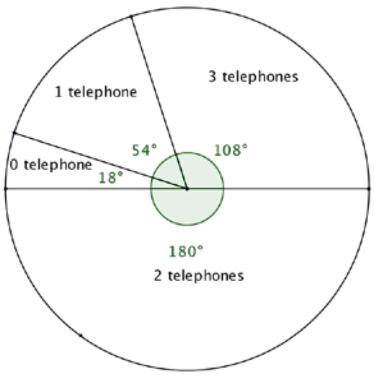

Circle graph</td><td>0,75

0.5 +
0.25</td></tr>
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