

المادة: الرياضيات الشهادة: المتوسطة نموذج رقم - ٥ - المدة : ساعتان	الهيئة الأكاديمية المشتركة قسم : الرياضيات	 المركز التربوي للبحوث والإنماء
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نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
 - يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (1.5points)

Given the number $a = \frac{1 + \sqrt{5}}{2}$.

- 1) Show that: $a^2 - a - 1 = 0$.
- 2) Prove that: $\frac{1}{a} = a - 1$.

II - (3points)

- 1)
 - a) Verify that $x^2 + 6x + 5 = (x + 3)^2 - 4$
 - b) Factorize $x^2 + 6x + 5$.
- 2) ABC is a triangle right at A, where $AC = 2x + 2$ and its area is $x^2 + 6x + 5$, (x is a positive real number).
 - a) Show that $AB = x + 5$.
 - b) Calculate, in terms of x, BC^2 .
- 3) Determine x if the area of triangle ABC equals 12 units of area.

III- (2.5points)

A survey about the content of a website was organized. Some visitors of the website were asked to rate the content using a 5-unit mark.

The table below shows the different opinions of the visitors.

Mark	1	2	3	4	5
Frequency	4	10	14	12	10

- 1) Calculate the average mark of the obtained marks.
- 2) The owners of the website will be satisfied if at least 65% of the visitors give a mark greater than or equal to 3. Will the owners of the website be satisfied? Explain your answer.
- 3) Construct the bar graph of this distribution.

IV-(2.5points)

To transport goods, Mr. Jadhas the opportunity to address two transport companies: A and B. The offers presented by the two companies are given below.

Offer of Company A: 2 000 LL per kilometer.

Offer of Company B: 210 000 LL down payment and 1 500 LL per kilometer.

- 1) Calculate the amount to be paid for a distance of 100 km by the two offers.
- 2) Let x be the number of kilometers traveled. Denote by y_1 the price charged by Company A and by y_2 the price charged by Company B.
Express y_1 and y_2 in terms of x.

- 3) Calculate the number of kilometers traveled for which the prices charged in the two offers are the same. What is, then, the price to pay?
- 4) Jad chooses offer A and pay 500 000 LL. Does he choose the more advantageous offer?

V- (5.5 points)

ABE is an isosceles triangle of vertex B such that $BA = BE = 6$ cm and $\widehat{ABE} = 140^\circ$. The circle (C) with diameter [BE] and center O intersects the line (AB) at point F. Let I be the midpoint of [AE].

- 1) Draw a figure.
- 2) What is the nature of triangle BEF? Justify your answer.
- 3) Prove that I is on (C).
- 4) Calculate an approximate value of BF to the nearest millimeter.
- 5) a- Show that the triangles ABI and AEF are similar. Deduce that $AB \times AF = 2AI^2$.
b- Then calculate AE.
- 6) The two lines (BI) and (EF) intersect at H. Let (d) be the parallel through B to (AL). The line (d) intersects (AE) and (EF) at G and L respectively.
 - a) Prove that (LG) is tangent to (C).
 - b) Calculate the ratio $\frac{EL}{EH}$.

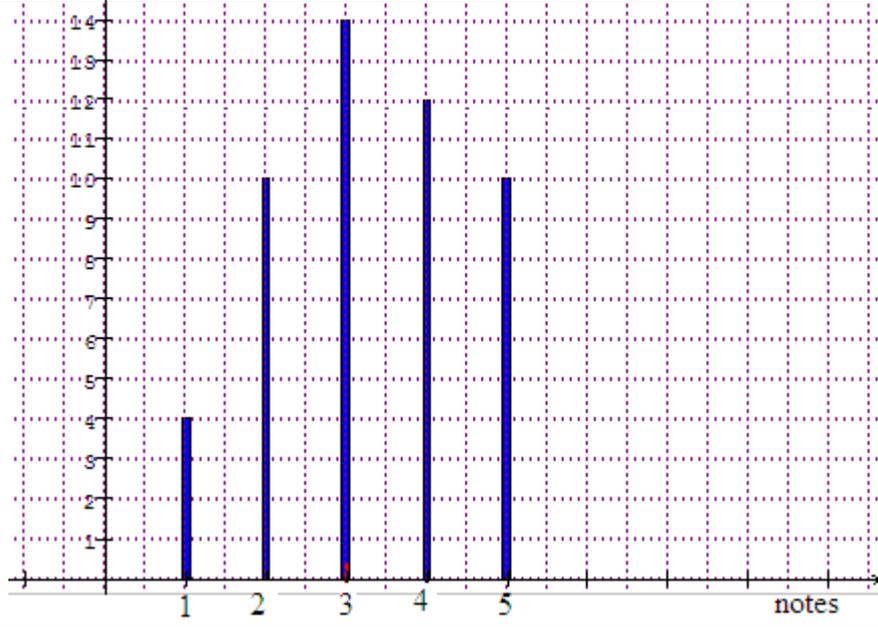
VI- (5 points)

In an orthonormal system of axes $x'Ox$ and $y'Oy$, Consider the points $A(4 ; 2)$ and $B(0 ; 4)$, Let (d) the line through B and perpendicular to (OA).

- 1) Plot the points A, B and draw (d).
- 2) a) show that the triangle AOB is isosceles.
b) Prove that $y = -2x + 4$ is the equation of (d).
- 3) Let H be the orthocenter of the triangle OAB.
 - a) Verify that $y_H = 2$.
 - b) Calculate x_H .
- 4) Denote by E the meeting point of (OH) and (AB), and F that of (d) and $x'Ox$.
 - a) Calculate the coordinates of F.
 - b) Prove that the 2 triangles OBF and EOB are similar. Calculate the similarity ratio.
- 5) Let (C) be the circle circumscribed about the triangle BOF. Denote by (T) the tangent at B to (C). The line (T) intersects the x-axis at G.
 - a) Show that (T) is parallel to (OA)
 - b) Write an equation of (T).

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أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

Question I		
Answers		Note
2	$a+1 = \frac{3+\sqrt{5}}{2}$; $a^2 = \frac{3+\sqrt{5}}{2}$ then $a+1 = a^2$ therefore $a^2 - a - 1 = 0$	1
3	$\frac{1}{a} = \frac{2}{1+\sqrt{5}} = \frac{1-\sqrt{5}}{-2}$ $a-1 = \frac{1-\sqrt{5}}{-2}$	0.5
Question II		
1.a	$(x+3)^2 - 4 = x^2 + 6x + 9 - 4 = x^2 + 6x + 5$	0.5
1.b	$x^2 + 6x + 5 = [x+3-2][x+3+2] = (x+1)(x+5)$	0.5
2.a	Area of ABC = $\frac{AB \times AC}{2}$, $(x+1)(x+5) = \frac{2(x+1) \cdot AB}{2}$ then $AB = x+5$	0.75
2.b	$BC^2 = (2x+2)^2 + (x+5)^2 = 5x^2 + 18x + 29$	0.5
3	$(x+3)^2 - 4 = 12$ then $(x+3)^2 = 16$, $x+3 = 4$ or $x+3 = -4$ then $x = 1$, $x = -7$ impossible.	0.75
Question III		
1	$\bar{x} = \frac{1 \times 4 + 2 \times 10 + 3 \times 14 + 4 \times 12 + 5 \times 10}{50} = 3,28$	0.75
2	$14+12+10 = 36$ the frequency ≥ 3 . $\frac{36}{50} \times 100 = 72\%$. $72\% \geq 65\%$ true.	0.75
3		1
Question IV		
1	A : Sum = $2000 \times 100 = 200\,000$ LL B : Sum = $1500 \times 100 + 210\,000 = 370\,000$ LL	0.5

2	$y_1 = 2000x$, $y_2 = 1500x + 210000$	0.5
3	$2000x = 1500x + 210000$ alors $x = 420$ km et le prix payé = $2000 \times 420 = 820000$ L.L	0.75
4	Pour A : $500000 = 2000x$ alors $x = 250$ km If he choice B, sum = $1500 \times 250 + 210000 = 585000$ LL. Yes Jad do good choice.	0.75

Question V

1		0.5
2	BFE right angled triangle (inscribed in half circle).	0.5
3	I midpoint of [AE] then [BI] perpendicular bisector in ABE isosceles , then BIE is right at I, then inscribed in half circle of diameter [BE]	0.5
4	ABE isosceles then : $\widehat{BAE} = \widehat{AEB} = (180 - 140) \div 2 = 20^\circ$ $\widehat{EBF} = 40^\circ$ (exterior). $\cos \widehat{EBF} = \frac{BF}{BE}$ alors $BF = BE \times \cos(40^\circ) = 6 \times \cos(40^\circ) = 4,59$ cm	1
5.a	$\widehat{BFE} = \widehat{BIA} = 90^\circ$ A common angle . $\frac{AB}{AE} = \frac{AI}{AF}$ then : $AB \times AF = AI \times AE = AI \times 2 AI = 2 AI^2$.	0.5 0.75
5.b	In EBI right at B we have: $AF = 6 + 4.59 = 10,59$ cm $AB \times AF = 2 AI^2$ we calculate AI then $AE = 2AI$.	
6.a	B orthocenter of AEH then [EB] third altitudethen (EB) perpendicular to(AH) then to (GL).	1,25
6.b	Thales : $\frac{EL}{EH} = \frac{EG}{EA}$ but $EG = \frac{EB}{\cos 20} = 6.38$ then $\frac{EL}{EH} = \frac{6.38}{11.26}$	0,5

Question VI

1		0,5
2.a	$AB = AO = \sqrt{20}$	0,5
2.b	Slope of(OA) = $\frac{1}{2}$, (d) perpendicular à (OA) then Slope of(d)= -2 B is on (d) because $4 = -2 \times 0 + 4$	0,5
3.a	(AH) altitude then perpendicular to (y'y) then $y_H = y_A = 2$	0,5
3.b	H is on (d) then $x_H = 1$	0,5
4.a	$y_F = 0$ and F is on (d) then $x_F = 2$	0,5
4.b	$\hat{O} = \hat{E} = 90$ and $E\hat{O}B = O\hat{B}F$ HOB isocèles triangle Ratio = $\frac{OB}{BF} = \frac{4}{\sqrt{20}}$	1
5.a	(T) and(OA) are parallel since they are perpendicular to (FB).	0,5
5.b	$a(T) = a(OA) = \frac{1}{2}$ and B on (T) then the equation is $y = \frac{1}{2}x + 4$.	0,5