مسابقة في تايضايرلا ةدام
المدة: ساعتان

الاسم:
الرقم:

ملاحظــة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (2 points)

In the table given below, only one among the proposed answers to each question is correct.
Write down the number of each question and give, with justification, the answer corresponding to it.

N°	Questions	Answers		
		a	**b**	**c**
1°	$\frac{4}{5} - \frac{3}{5} \times \frac{10}{6} =$	$-\frac{1}{5}$	$\frac{26}{25}$	$\frac{1}{3}$
2°	$3^{14} - 3^{12} =$	3^2	$3^{12} \times 8$	6
3°	x is an acute angle such that $\cos x = \frac{1}{3}$, then sin x =	$\frac{2}{3}$	$\frac{\sqrt{2}}{3}$	$\frac{2\sqrt{2}}{3}$
4°	An object costs 270LL. Its price is increased by 5%. The new price of the object is :	275 LL	270.05LL	283.5 LL

II- (1½ points)

Given the two numbers A and B :

$$A = \frac{3.6 \times 10^3 \times 10^{-5}}{9 \times 10^2} \quad \text{and} \quad B = \left(2 + \sqrt{5}\right)^2 + \sqrt{5}\left(1 + 2\sqrt{5}\right).$$

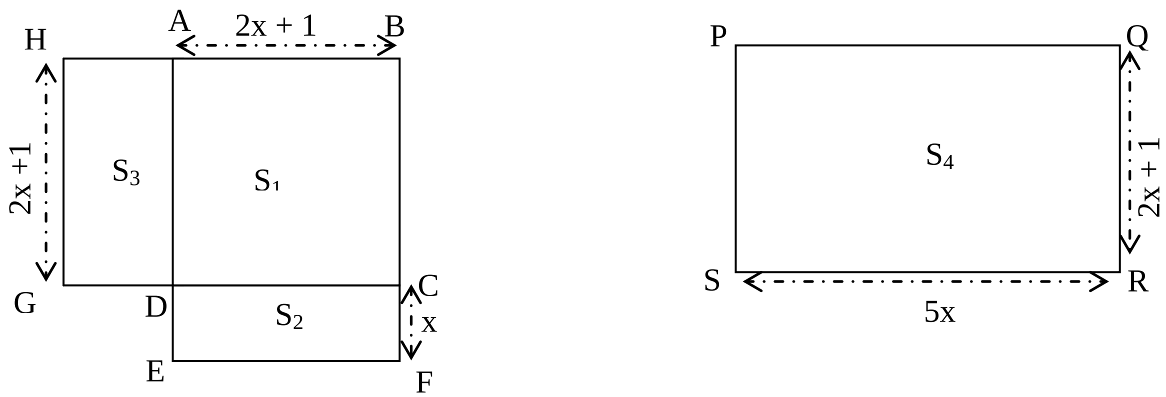
1) Write A in the form $a \times 10^n$ where a and n are two integers, then write A in the form of a decimal number.

2) Write B in the form $b + c\sqrt{5}$ where b and c are two integers.

III- (2½ points)

1) Consider the expression : $E(x) = 4x^2 - 1 + (2x+1)^2 + x(2x + 1)$.
Show that $E(x) = 5x\,(2x + 1)$.

2) In the figure below :



- x is a measure of length in centimeters and $2x - 1 > 0$.
- ABCD is a square of area S_1 .
- DCFE , HADG and PQRS are three rectangles of areas S_2 , S_3 and S_4 respectively.

a- Express S_1 and S_2 in terms of x .

b- Knowing that $S_1 + S_2 + S_3 = S_4$ and using the preceding results, calculate AH in terms of x .

IV- (2 points)

If we add 5 to each term of the irreducible fraction $\frac{x}{y}$, we get a fraction equal to $\frac{7}{8}$. If we subtract 3 from each term of this fraction, we get a fraction equal to $\frac{3}{4}$.

1) Show that the preceding information is translated into the following system of two equations with two unknowns :

$$\begin{cases} 8x - 7y = -5 \\ 4x - 3y = 3 \end{cases}$$

2) Solve this system, showing all the steps of calculation, and find the fraction $\frac{x}{y}$.

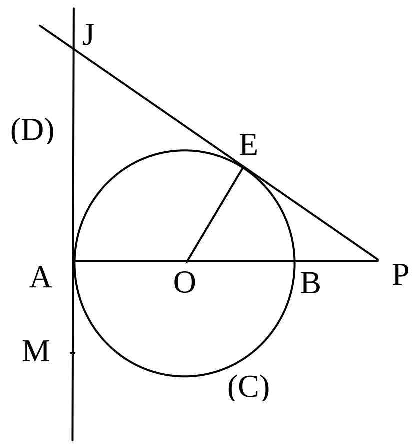
V- (6½ points)

Consider in an orthonormal system of axes $x'Ox$, $y'Oy$, the points A(– 3 , 6), B(5 , 2) and E(1,– 2) and the straight line (d) of equation $y = x - 3$.

1) Plot the points A , B and E.
2) Verify by calculation, that E and B are two points of straight line (d). Draw (d).
3) a- Write the equation of the straight line (AE). Deduce that the points E, A and O are collinear.
   b- Are the straight lines (d) and (AE) perpendicular? Justify your answer.
4) Designate by M and N the respective images (translates) of A and C by the translation of vector $\overrightarrow{EB}$, and designate by (D) the image (translate) of $(y'y)$ by the same translation.
   a- Prove that B, N and M are collinear.
   b- Calculate the coordinates of $\overrightarrow{EB}$.
   c- Calculate the coordinates of N .
   d- Draw (D) and find its equation.
5) a- Show that AEBM is a parallelogram and not a rectangle.
   b- The diagonals of AEBM intersect in J. Calculate the coordinates of J.

VI- (5½points)

In the opposite figure:



- (C) is a circle of center O and diameter [AB]
- OA = OB = 3 cm
- P is the point of [AB) such that OP = 5 cm
- E is a point of (C) such that PE = 4 cm
- (D) is the tangent at A to (C)
- M is a variable point on (D)
- (PE) cuts (D) in J.

1) Reproduce this figure. It will be used and completed by the remaining parts of this problem.
2) a- Prove that (PE) is tangent to (C) at E . Deduce that JE = JA.
   b- Calculate tan $O\hat{P}E$ and round to the nearest degree the angle $O\hat{P}E$.
3) Let JE = JA = x and JP = x + 4 where x is a measure of length in centimeters.
   a- Apply Pythagoras theorem to triangle APJ and calculate x.
   b- Deduce that triangle ABJ is a right isosceles triangle.
4) (JB) cuts (C) in a second point F. Prove that F is the midpoint of [JB] and that (FO) is the perpendicular bisector of [AB].
5) Let N be the midpoint of [MB].
   Find the locus of N when M moves on (D).

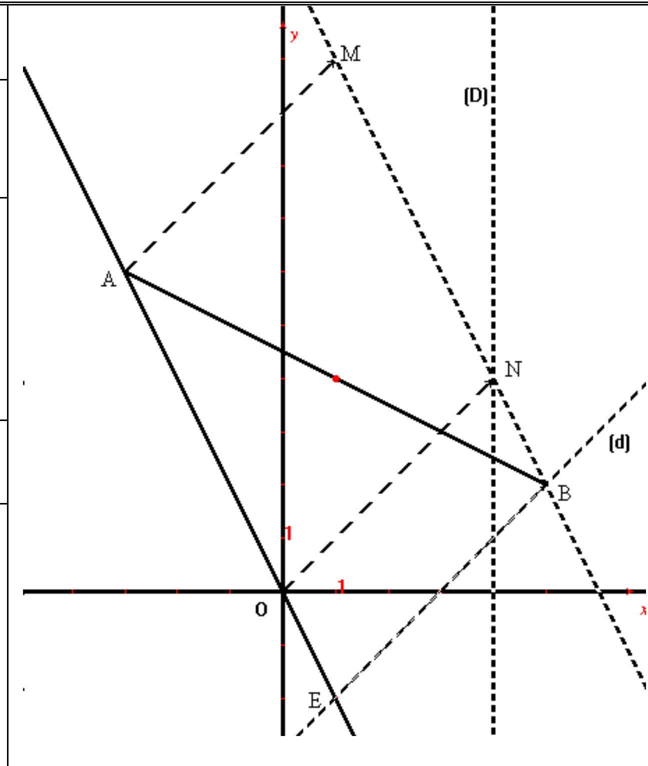
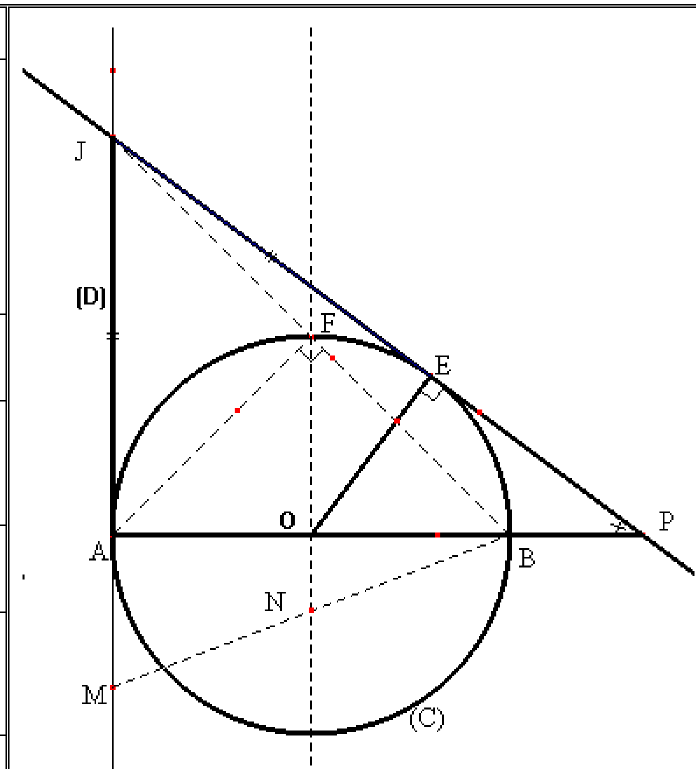
دورة سنة ٢٠٠٦ العادية

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة المتوسطة

توزيع علامات مسابقة الرياضيات

I	1)	$\frac{4}{5}-\frac{3}{5}\times\frac{10}{6}=-\frac{1}{5}$	½
	2)	$3^{14}-3^{12}=3^{12}\times 8$	½
	3)	$\sin^2 x+\cos^2 x=1\,;\sin^2 x=\frac{8}{9}\ ;\ \sin x=\frac{2\sqrt{2}}{3}$	½
	4)	Le nouveau prix: $270+\frac{270\times 5}{100}=283{,}5$ L.L	½
II	1)	$A=\frac{3{,}6\times 10^3\times 10^{-5}}{9\times 10^2}=\ldots=4\times 10^{-5}$ $A=0{,}00004$	¾
	2)	$B=\ldots=19+5\sqrt{5}$	¾
III	1)	$E(x)=\ldots..\ =5x(2x+1)$	1
	2) a-	$s_1=(2x+1)^2\ ;\ s_2=x(2x+1)$	¾
	2) b-	$s_1+s_2+s_3=s_4$ $(2x+1)^2+x(2x+1)+AH\cdot(2x+1)=5x(2x+1)$ $AH=2x-1$	¾
IV	1)	$\begin{cases}\frac{x+5}{y+5}=\frac{7}{8}\\ \frac{x-3}{y-3}=\frac{3}{4}\end{cases}\ ;\ \begin{cases}8x-7y=-5\\ 4x-3y=3\end{cases}$	1
	2)	$\begin{cases}8x-7y=-5\\ -8x+6y=-6\end{cases}\ ;\ y=11\ \text{et}\ x=9\ ;\ \frac{x}{y}=\frac{9}{11}$	1

V	1)		Placer A, B et E		½
	2)		(d): $y = x - 3$ -2=1-3 donc E est sur (d) 2=5-3 donc B est sur (d)		¾
	3)	a-	Equation de (AE) : y = ax + b $\begin{cases} 6 = -3a + b \\ -2 = a + b \end{cases}$; $a = -2$, $b = 0$ $y = -2x$, passe par O Alors A, E et O sont alignées.		1
		b-	pente de (d) . pente $(AE) = -2$ donc (d)n'est pas perpendiculaire à (AE)		½
	4)	a-	$\overrightarrow{AM} = \overrightarrow{EB}$ donc AEBM et un parallélogramme $\overrightarrow{ON} = \overrightarrow{EB}$ donc OEBN est un parallélogramme Alors B ; M et N sont alignées (ou…)		1
		b-	$\overrightarrow{EB}$ (4 ; 4)		½
		c-	$\overrightarrow{ON} = \overrightarrow{EB}$ alors N (4 ; 4)		½
		d-	x = 4 (D)		¾
	5)	a-	$\overrightarrow{AM} = \overrightarrow{EB}$; AEBM est un parallélogramme (AE) n'est pas perpendiculaire à (EB) alors AEBM n'est pas un rectangle.		½
		b-	J milieu de [AB] ; $J(1;4)$		½
VI	1)		figure		¼
	2)	a-	$PE^2 + OE^2 = 16 + 9 = 25$ $OP^2 = 25$ donc OPE est un triangle rectangle en E. (PE) perpendiculaire à OE alors (PE) est tangente au cercle. JE = JA (…)		1
		b-	$\tan O\hat{P}E = \frac{3}{4}; O\hat{P}E = 36°,8 \approx 37°$		¾
	3)	a-	$PJ^2 = AJ^2 + AP^2$; $(x+4)^2 = x^2 + 64$; $x = 6$		1
		b-	$AB = AJ = 6$; $J\hat{A}B = 90°$ JAB est un triangle rectangle isocèle		½
	4)		$A\hat{F}B = 90°$; [AF] hauteur à la fois médiane , donc F milieu de [BJ] (FO) médiatrice de [AB].		1
	5)		N décrit la médiatrice de [AB]		1