المادة: الرياضيات الشهادة: الثانوية العامة الفرع: علوم الحياة نموذج رقم -٢- المدّة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- **(4 points).**

The following statements are true. Justify.

1) The complex plane refers to a direct orthonormal system $(O;\vec{u},\vec{v})$. Consider the distinct points A, B and C with affixes a, b and c so that $\frac{c-a}{b-a}$ = 2i; therefore, A is on the circle with diameter [BC].

2) If $\frac{\pi}{2}$ is an argument of z, then $|i+z|$ = 1 + $|z|$.

3) If z = 3 + 3i$\sqrt{3}$ then z^3 is pure imaginary.

4) If z = $e^{i\theta}$, then $z^2 + \frac{1}{z^2}$ is real.

5) $|i\bar{z}+1| = |z+i|$.

II- (4 points)

Given, in the space of an orthonormal system $(\mathbf{O};\ \vec{\mathbf{i}},\vec{\mathbf{j}},\vec{\mathbf{k}})$, the point A(2 ; 1 ; 5) and the two straight lines (d) and (d') defined by: (d) $\begin{cases} x = 2m+4 \\ y = 2m+1 \\ z = -3m-5 \end{cases}$ and (d′) $\begin{cases} x = t+2 \\ y = 2t-1 \\ z = -2t+6 \end{cases}$, where m and t are real numbers. Let (P) be the plane determined by A and (d’).

1) **a)** Show that A is neither on (d) , nor on (d’).
   b) Prove that (d) and (d’) are skew.

2) **a)** Show that 2x + y + 2z – 15 = 0 is an equation of the plane (P).
   b) Prove that (d) is parallel to (P).

3) Let (Δ) be the line through A and parallel to (d).
   a) Write a system of parametric equations for (Δ).
   b) Find the coordinates of B, the meeting point of (Δ) and (d’) .

4) Let E be the orthogonal projection of A on (d’).
   a) Calculate the coordinates of E.
   b) Calculate the area of the triangle AEB.

5) Let M be a point on (d); calculate the volume of tetrahedron MABE .

III- (4 points)

U_1 and U_2 are two given urns such that:

U_1 contains 10 balls: 6 red and 4 yellow.

U_2 contains 10 balls: 5 red, 4 black and 1 green.

A fake coin is given such that the probability of having a head is three times more than that of having a tail.

The coin is tossed.

- If we get a tail, we select, randomly and **simultaneously** two balls from urn U_1.
- If we get a head, we select at random two balls from U_2 **one after the other with replacement**.

Consider the following events:

U_1: "The selected urn is U_1."

R: "The selected balls are red."

1) Show that $P(U_1) = \frac{1}{4}$.
2) Calculate $P(R / U_1)$ and $P(R \cap U_1)$. Deduce that $P(R) = \frac{13}{48}$.
3) The two selected balls are red. Calculate the probability that they come from U_1.
4) Let X be the random variable that represents the number of red balls obtained. Determine the probability distribution of X.

IV- (8 points)

Part A

Consider the function g defined over $]1, +\infty[$ as: $g(x) = x(\ln x)^2 - e$.

1) Determine the limit of g(x) as $x \to 1$ and $x \to +\infty$.
2) a) Set up the table of variations of g.

   b) If $x > e$, prove that $g(x) > 0$.

Part B

Let f be a function defined over $]1, +\infty[$ as $f(x) = e\,\frac{\ln x - 1}{\ln x} - x$. Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$ and (d) the line with equation $y = e - x$.

1) **a)** Find $\lim_{x\to 1} f(x)$; deduce an asymptote to (C).

   b) Prove that (d) is an asymptote to (C).

   c) Prove that (C) is below (d).
2) **a)** Prove that $f'(x) = \frac{-g(x)}{x\ln^2 x}$.

   b) Set up the table of variations of f.

   c) Draw (C).
3) **a)** For $1 < x \leq e$, prove that f has an inverse function h. Determine the domain of h.

   b) Draw the curve (C') of h in the same system as that of (C).
4) Let (Δ) be the line with equation $y = -x - e$.

   a) Determine the coordinates of point A, the meeting point of (C') and (Δ).

   b) Write an equation of (T), the tangent at A to the curve (C').

   c) Solve graphically the inequality $h(x) + e > -x$.

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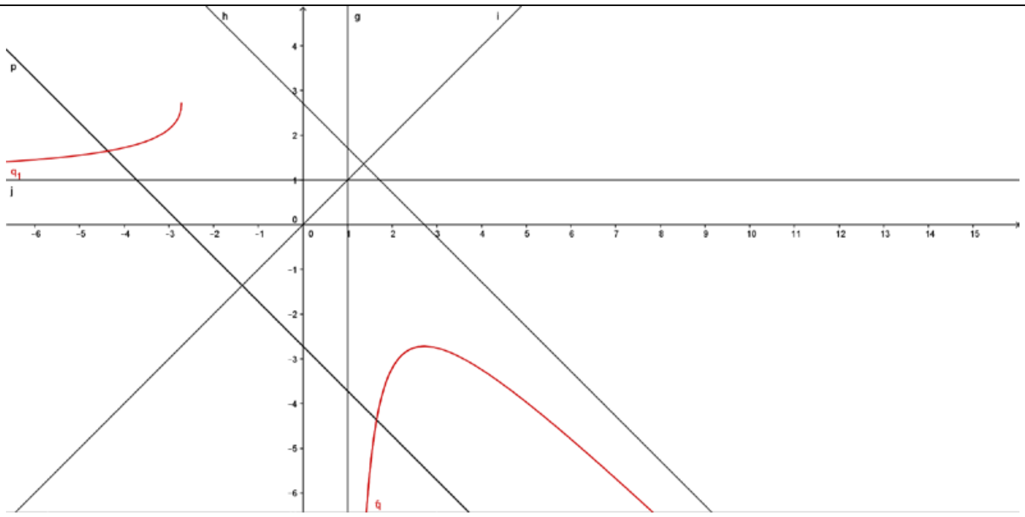
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

Question I (4 points)		Mark
1	$\frac{c-a}{b-a}=2i$ then $\frac{z_{\overrightarrow{AC}}}{z_{\overrightarrow{AB}}}=\frac{\pi}{2}$ and $\left(\overrightarrow{AB},\overrightarrow{AC}\right)=\frac{\pi}{2}$. Therefore, A is on the circle with diameter [BC].	0.75
2	$\arg(Z)=\frac{\pi}{2}$ Then z = ib with b >0. i+z =i(1+b) and $\lvert i+z\rvert=1+b$. $1+\lvert z\rvert=1+\lvert ib\rvert=1+b$.	0.75
3	Z = 3 + i $\sqrt{3}=2\sqrt{3}e^{i\frac{\pi}{6}}$ and $z^3=24e^{i\frac{\pi}{2}}$ pure imaginary.	0.75
4	Z = $e^{i\theta}$ then $z^2+\frac{1}{z^2}=e^{2i\theta}+e^{-2i\theta}=2\cos(2\theta)$ (real).	0.75
5	$\left\lvert i\bar{z}+1=\lvert i(a-ib)+1\rvert=\lvert 1+b+ia\rvert=\sqrt{(1+b)^2+a^2}\right\rvert$ $\lvert z+i\rvert=\lvert a+i(b+1)\rvert=\sqrt{a^2+(b+1)^2}$.	1

Question II (4 points)		Mark
1.a	A is not on (d) and A is not on (d’).	**0.5**
1.b	(d) and (d’) are neither parallel nor intersecting .	**0.5**
2.a	A and (d’) satisfy the equation: 2x + y +2z −15 = 0 (P).	**0.5**
2.b	$\overrightarrow{V_d}(2,2,-3).\overrightarrow{n_p}(2,1,2)=0$ and (d) is not on (P) then (d) is parallel to (P).	**0.5**
3.a	(Δ): x = 2k+2 , y = 2k+1 , z=−3k+5.	**0.5**
3.b	B(4,3,2) for k=1 and t =2.	
4.a	For $E\left(\frac{8}{3};\frac{1}{3};\frac{14}{3}\right)$ with $\overrightarrow{AE}.\overrightarrow{U_{(d')}}$ =0	**0.5**
4.b	Area $=\frac{AE.EB}{2}=2u^2$	**0.5**
5	V= $\frac{1}{6}\left\lvert\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AE})\right\rvert$ = constant because (d) is parallel to (P).	**0.5**

Question III (4 points)		Mark
1)	$P(U_1) + P\left(\overline{U_1}\right) = 1$ and $P\left(\overline{U_1}\right) = 3P(U_1)$; then $P\left(\overline{U_1}\right) = \frac{3}{4}$ and $P(U_1) = \frac{1}{4}$	0.5
2)	$P(R / U_1) = \frac{1}{3}$ $P(R \cap U_1) = P(R / U_1) \times P(U_1) = \frac{1}{12}$ and $P\left(R \cap \overline{U_1}\right) = P\left(R / \overline{U_1}\right) \times P\left(\overline{U_1}\right) = \frac{3}{16}$ $P(R) = P(R \cap U_1) + P\left(R \cap \overline{U_1}\right) = \frac{13}{48}$	0.25 0.5 0.5 0.5
3)	$P(U_1 / R) = \frac{4}{13}$	0.5
4)	$X_\Omega = \{0 , 1 , 2\}$; $P(X = 0) = \frac{53}{240}$; $P(X = 1) = \frac{61}{120}$; $P(X = 2) = \frac{13}{48}$	0.25 0.5 0.5

<table>
<tr><th colspan="2">Question IV (8 points)</th><th>Mark</th></tr>
<tr><td colspan="3"><u>Part A</u></td></tr>
<tr><td>1</td><td>$x \to 1, g(x) \to -e$
$x \to +\infty, g(x) \to +\infty$</td><td>0.5</td></tr>
<tr><td>2.a</td><td>g’(x) = $(\ln x)^2 + 2x(\ln x)\frac{1}{x}$ = lnx(lnx+2).
<table>
<tr><td>x</td><td>1</td><td></td><td>$+\infty$</td></tr>
<tr><td>g’(x)</td><td>0</td><td>+</td><td></td></tr>
<tr><td>g(x)</td><td>-e</td><td>↗</td><td>$+\infty$</td></tr>
</table></td><td>0.5</td></tr>
<tr><td>2.b</td><td>g(e)=0, then g(x) >0 for x > e.</td><td>0.5</td></tr>
<tr><td colspan="3"><u>Part B</u></td></tr>
<tr><td>1.a</td><td>$x \to 1, f(x) \to -\infty$ then x=1 is a vertical asymptote .
$x \to +\infty, f(x) \to -\infty$.</td><td>0.25</td></tr>
<tr><td>1.b</td><td>$\lim\limits_{x \to +\infty} \left[f(x) - e + x \right] = \lim\limits_{x \to +\infty} e\left[\frac{\ln x - 1}{\ln x} - 1\right] = 0$
Then (d) is an asymptote to (C).</td><td>0.5</td></tr>
<tr><td>1.c</td><td>f(x) -y = $e\left(\frac{-1}{\ln x}\right)$ <0 then (C) is below (d).</td><td>0.5</td></tr>
<tr><td>2.a</td><td>f ’(x)= $e\left(\frac{1}{x\ln^2 x}\right) - x = \frac{-g(x)}{x\ln^2 x}$.</td><td>0.75</td></tr>
<tr><td>2.b</td><td>
<table>
<tr><td>x</td><td>1</td><td></td><td>e</td><td></td><td>$+\infty$</td></tr>
<tr><td>f’(x)</td><td></td><td>+</td><td>0</td><td>−</td><td></td></tr>
<tr><td>f (x)</td><td>$-\infty$</td><td>↗</td><td>-e</td><td>↘</td><td>$-\infty$</td></tr>
</table></td><td>0.5</td></tr>
</table>

2.c		**1**
3.a	For x $\in$]1;e], f is continuous and strictly increasing, then f has an inverse function h .Dh=]-∞,-e].	**0.5**
3.b	Plot (C’)	**0.5**
4.a	(Δ) is the symmetric of $(\Delta)$with respect to y= x .Find $(\Delta)\cap(C)$: $e(\frac{\ln x-1}{\ln x})-x=-x-e$. lnx $=\frac{1}{2}$ and x = $\sqrt{e}$ $(\Delta)\cap(C)$= A’ $(\sqrt{e},-\sqrt{e}-e)$. $(\Delta)\cap(C')$= A($-\sqrt{e}-e,\sqrt{e}$)	**0.5**
4.b	f ’(x) = $\frac{-g(x)}{x\ln^2 x}$;f ’($\sqrt{e}$)= $1-4\sqrt{e}$. Slope of (T) = $\frac{1}{1-4\sqrt{e}}$. (T) : y - $\sqrt{e}=\frac{1}{1-4\sqrt{e}}(x+\sqrt{e}+e)$.	**1**
4.c	h(x) > $-$x$-$e. Consider the part of (C’) that is above (Δ) then x $\in]-\sqrt{e}-e,-e]$.	**0.5**