

عدد المسائل : ستة	مسابقة في مادة الرياضيات المدة ساعتان	الاسم : الرقم :
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ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة .

I- (3 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question ,and give with justification, its corresponding answer .

N ⁰	Question	Proposed answers		
		a	b	c
1	In an orthonormal system, the two lines with equations $y = 2x + 3$ and $2y + x = 1$ are...	intersecting at the point (1,5)	perpendicular	parallel
2	The original price of an article is 30 000 LL ,and its price after the discount is 27 600 LL . The percentage of discount is...	10 %	18 %	8 %
3	A triangle ABC is right at B. If $AB=3$ and $\angle ACB = 40^\circ$, then $AC = \dots$	$\frac{3}{\sin 40^\circ}$	$3\sin 40^\circ$	$\frac{\sin 40^\circ}{3}$
4	The natural numbers, solutions of $3x-1 < 8$ are...	1 ; 2 ; 3	0 ; 1 ; 2 ; 3	0 ; 1 ; 2
5	A triangle ABC is right at A and M is the midpoint of [BC]. If $AM = AB = 6$ then $AC = \dots$	12	$6\sqrt{3}$	$9\sqrt{3}$
6	ABCD is a parallelogram and E is the translate of D under the translation with vector $\overrightarrow{BA}$. Therefore...	C is the midpoint of [DE]	$\overrightarrow{DB} + \overrightarrow{DE} = \overrightarrow{DA}$	$\overrightarrow{DE} + \overrightarrow{DA} = \overrightarrow{DB}$

II-(2 points)

Consider the three numbers A , B and C :

$$A = (\sqrt{5} + 1)^2 + (\sqrt{5} - 1)^2 ; B = \frac{3}{4} + \frac{5}{4} \times \frac{7}{15} \text{ and } C = \frac{(10\sqrt{3})^4}{25 \times 10^3 \times 3 + 10^3 \times 6 \times 37.5} .$$

In the following questions , the steps of calculation must be shown

- 1) Prove that A is a natural number.
- 2) Write B as a fraction in its simplest form.
- 3) Prove that C is a decimal number.

III- (2 points)

In what follows, are the scores of a student in five tests: 10 ; 8 ; 13 ; x and y.

The difference between x and y is 7. The average (mean) of these five scores is 12.

- 1) Write a system of two equations with two unknowns modeling the given situation .
- 2) Solve the obtained system .

IV- (3 points)

Consider the expressions: $A(x) = 4x^2 - 9$ and $B(x) = (2x - 3)^2 - (2x - 3)(x - 5)$.

- 1) Factorize $A(x)$.
- 2) a- Verify that $B(x) = (2x - 3)(x + 2)$.
b- Solve the equation $B(x) = 0$.
- 3) Consider the expression $F(x) = \frac{A(x)}{B(x)}$.
 - a. For what values of x , $F(x)$ is defined ?
 - b. Simplify $F(x)$ and solve $F(x) = 3$.
 - c. Calculate $F(\sqrt{5})$ and write the answer in the form $a - b\sqrt{5}$. (a and b are natural numbers)

V- (5 points)

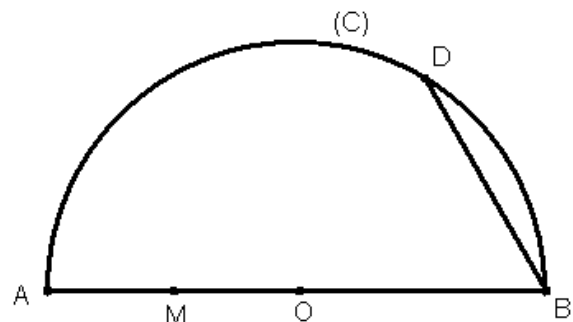
In an orthonormal system of axes $x'Ox$, $y'Oy$, where the unit of length is the centimeter, consider the points $A(1; -2)$, $B(2; 1)$, $C(5; 0)$ and the line (d) with equation $y = 3x - 5$.

- 1) a. Verify that (d) is passing through A and B .
b. Plot the points A , B , C and draw (d) .
- 2) a. Determine the equation of the line (BC) .
b. Calculate the lengths AB and BC .
c. Prove that ABC is a right isosceles triangle with vertex B .
- 3) Let (G) be the circle circumscribed about the triangle ABC and D is the point defined by $\overrightarrow{BD} = \overrightarrow{AC}$.
 - a. Calculate the coordinates of D .
 - b. Prove that (DB) is tangent to (G) .
- 4) (d') is the translate of (d) under the translation with vector $\overrightarrow{AC}$. Find the equation of (d') .

VI- (5 points)

Consider a semicircle (C) with centre O and radius R . $[AB]$ is the diameter of (C) and D is a point on (C) so that $BD = R$.

Let M be the midpoint of $[OA]$. The perpendicular bisector of $[OA]$ intersects $[AD]$ at E , (BD) at F and (C) at K .



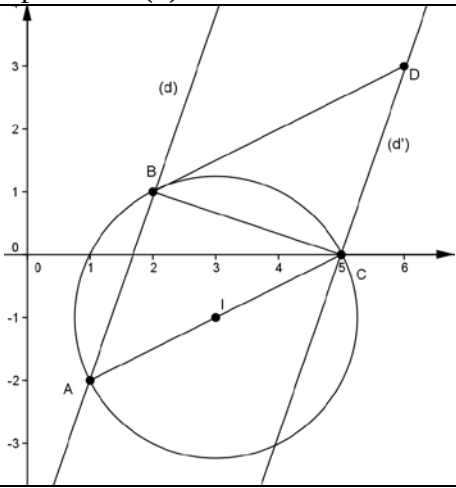
- 1) Reproduce and complete the figure.
- 2) Calculate the angles of the triangle ABD , then calculate AD in terms of R .
- 3) a. Prove that the two triangles ADB and FMB are similar.
b. Calculate BF in terms of R .
- 4) (BE) intersects (AF) at J . Show that J is on (C) .
- 5) Prove that the points B , M , J and F are on the same circle whose center and radius should be determined.
- 6) a. Show that the triangle OKA is equilateral.
b. Prove that (OK) passes through the midpoint of $[AF]$.

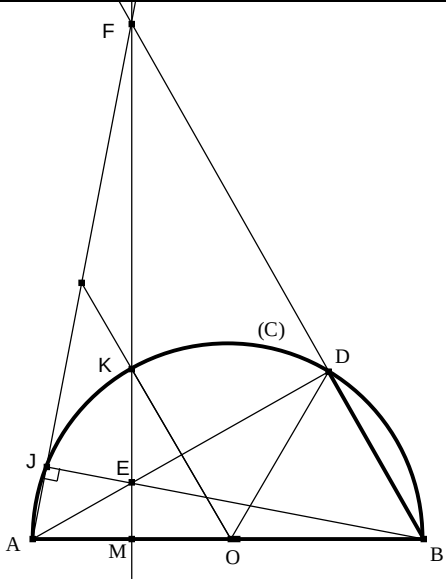
الاسم:
الرقم:مسابقة في مادة الرياضيات
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مشروع معيار التصحيح

	Part of the Q	Answer	Mark
I (3pts)	1	$(2)(-0.5) = -1$, therefore (b)	0.5
	2	$30\,000 - 27\,600 = 2400 = \frac{30\,000 \times x}{100}$; $x=8$ (8 %), therefore (c)	0.5
	3	$\frac{3}{\sin 40^\circ}$ therefore (a)	0.5
	4	0;1;2 therefore (c)	0.5
	5	ABM is equilateral, then $BC=12$ and $AC = \frac{12\sqrt{3}}{2} = 6\sqrt{3}$. therefore (c)	0.5
	6	DEAB is a parallelogram, $\overline{DB} + \overline{DE} = \overline{DA}$ therefore (c)	0.5
II (2pts)	Part of the Q	Answer	Mark
	1	$A=12$	0.75
	2	$B = \frac{4}{3}$	0.5
	3	$C = 0.3$	0.75
III (2pts)	Part of the Q	Answer	Mark
	1	$\frac{x+10+8+13+y}{5} = 12$; so, $x + y + 31 = 60$ Thus , $x + y = 29$ The system is $\begin{cases} y = x + 7 \\ x + y = 29 \end{cases}$	1.25
	2	$x = 11$, so $y = 18$.	0.75

	Part of the Q	Answer	Mark
IV (3pts)	1	$A(x) = (2x - 3)(2x + 3)$	0.5
	2.a	$B(x) = (2x - 3)(x + 2)$.	0.5
	2.b	$B(x) = 0$ $x = -2$ or $x = \frac{3}{2}$	0.5
	3. a	$F(x)$ is defined $x \neq -2$ and $x \neq \frac{3}{2}$	0.25
	3.b	$F(x) = \frac{2x+3}{x+2}$ $F(x) = 3$; $x = -3$.	0.75
	3.c	$F(\sqrt{5}) = \frac{2\sqrt{5}+3}{2+\sqrt{5}} = 4 - \sqrt{5}$.	0.5

V (5pts)	1. a	A and B are two points of (d)	0.5
	1. b		0.75
	2. a	$(BC): y = -\frac{1}{3}x + \frac{5}{3}$	0.75
	2. b	$AB = \sqrt{10}$; $BC = \sqrt{10}$	0.75
	2. c	$AB = \sqrt{10}$; $BC = \sqrt{10}$ and (AB) is perpendicular to (BC) then ABC is a right isosceles triangle at B.	0.5
	3.a	$\overline{AC}(4; 2); \overline{BD}(x; y); \overline{BD}(x - 2; y - 1)$ then $D(6; 3)$	0.5
	3.b	(BD) is perpendicular to (IB)	0.75
	4	$(d'): y = 3x - 15$	0.5

VI (5pts)	1	 Figure	0.5
	2	$\angle ADB = 90^\circ$, $\angle ABD = 60^\circ$, $\angle BAD = 30^\circ$: $AD = R\sqrt{3}$.	1
	3.a	The two triangles are right, and angle B is common	0.5
	3.b	$BF = 3R$	0.5
	4	(BE) is the third altitude in the triangle AFB, then ABJ is a right triangle at J, then J belongs to the circle	0.5
	5	$\angle FJB = \angle FMB = 90^\circ$ B, M, J and F are on the same circle. with center r midpoint of [BF], radius = $\frac{3R}{2}$	0.75
	6.a	AKO is an equilateral triangle since $KA = KO = OA = R$.	0.5
	6.b	(OK) parallel to (BF), O midpoint of [AB], (OK) passes through the midpoint of [AF].	0.75

