

عدد المسائل: ثلاث	مسابقة في مادة الرياضيات	الاسم: الرقم:
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ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (5 points)

1) Solve the following system:
$$\begin{cases} x + y = 20 \\ 30x + 25y = 560 \end{cases}$$

2) Fadi bought a box of 20 bottles of juice.

The price of this box is 56 000 LL. The box contains pineapple bottles for 3 000 LL each and orange bottles for 2 500 LL each.

a- How many bottles of each type did Fadi buy ? Justify.

b- If the price of each bottle of pineapple juice is subject to 20% discount, and that of the orange bottle remains the same. What is then the price of one box?

II- (5 points)

The following table represents the distribution of 80 employees of a certain factory according to their service years.

	Less than 10 years in service	Between 10 and 20 years in service	More than 20 years in service
Women	10	8	12
Men	14	16	20

1) One employee of this factory is randomly interviewed.

Consider the following events :

S: « The interviewed employee has less than 10 years in service ».

M: « The interviewed employee is a man ».

a- Calculate the following probabilities: $P(S)$, $P(M)$, $P(M \cap S)$ and $P(M/\bar{S})$.

b- The interviewed employee is a man. What is the probability that he has less than 10 years in service?

2) Two employees are interviewed one after another.

Consider the event :

A: « the two interviewed employees have more than 20 years in service ».

Show that $P(A) = \frac{62}{395}$.

III- (10 points)

The following table is the table of variations of a function f defined over $\mathbb{R} - \{1\}$.

Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

x	$-\infty$	0	1	2	$+\infty$						
f'(x)	+	0	-	-	0	+					
f(x)	$-\infty$	$\nearrow$	0	$\searrow$	$-\infty$	$+\infty$	$+\infty$	$\nwarrow$	4	$\nearrow$	$+\infty$

A– For each of the following statements, answer True or False and justify.

- 1) If $x < 0$, then $f(x) > 0$.
- 2) $f\left(\frac{1}{2}\right) < f\left(\frac{3}{4}\right)$.
- 3) $f'(-0.5) > f'(0.5)$.
- 4) The x -axis is tangent to the curve (C) at the origin O.

B- In what follows, let $f(x) = x + 1 + \frac{1}{x-1}$

- 1) Show that $f'(x) = \frac{x^2 - 2x}{(x-1)^2}$.
- 2) Write an equation of (T), the tangent to the curve (C) at its point A with abscissa $x = -2$.
- 3) Verify that the line (d) with equation $y = x + 1$ is an asymptote to (C).
- 4) Draw (d) and (C).
- 5) Show that the equation $f(x) = -2$ has two distinct solutions.

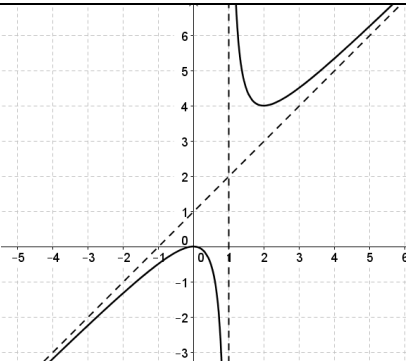
I- (5 points)

Q	Correction	Grade
1	$x = 12$; $y = 8$	1.5
2-a	let x be the number of pineapple bottles and y the number of orange bottles : $\begin{cases} x + y = 20 \\ 3000x + 2500y = 56000 \end{cases}$ $x = 12$; $y = 8$ 12 pineapple bottles and 8 orange bottles .	2
2-b	the price of one pineapple bottle is : $3000 - 3000 \times \frac{20}{100} = 2400$ LL the price of one orange bottle is : 2500 LL the price of one box is : $12 \times 2400 + 8 \times 2500 = 48800$ LL	1.5

II- (5 points)

Q	Correction	Grade
1-a	$P(M) = \frac{24}{80} = \frac{3}{10}$; $P(H) = \frac{50}{80} = \frac{5}{8}$; $P(H \cap M) = \frac{14}{80} = \frac{7}{40}$; $P(H / \overline{M}) = \frac{36}{56} = \frac{9}{14}$	2
1-b	$P(M / H) = \frac{14}{50} = \frac{7}{25}$	1.5
2	$P(A) = \frac{32}{80} \times \frac{31}{79} = \frac{62}{395}$	1.5

III- (10 points)

Q	Correction	Grade
A-1	False because if $x < 0$, then $f(x) \in]-\infty; 0[$	1
A-2	False because on $[0; 1[$ f is strictly decreasing $f\left(\frac{1}{2}\right) > f\left(\frac{3}{4}\right)$	1
A-3	True because for $x < 0$ we have $f'(x) > 0$ and for $x > 0$ we have $f'(x) < 0$	1
A-4	True because $f(0) = 0$ and $f'(0) = 0$	1
B-1	$f'(x) = \frac{x^2 - 2x}{(x-1)^2}$	1
B-2	$y = f'(-2)(x+2) + f(-2) = \frac{8}{9}x + \frac{4}{9}$	1
B-3	$\lim_{x \rightarrow +\infty} [f(x) - (x+1)] = \lim_{x \rightarrow +\infty} \frac{1}{x-1} = 0$ similarly for $x \rightarrow -\infty$.	1
B-4		1.5
B-5	the line of equation $y = -2$ intersect the curve (c) in two distinct points or by calculation.	1.5

