


المادة: الرياضيات الشهادة: الثانوية العامة - فرع الاجتماع والاقتصاد نموذج رقم -1- المدة : ساعتان	الهيئة الأكاديمية المشتركة قسم : الرياضيات	 المركز العربي للبحوث والتطوير
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نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي 2016-2017 وحتى صدور المناهج المطورة)

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (4 points)

The table below shows the VAT on cloths y_i , in the last 6 years in a certain country

Year	2010	2011	2012	2013	2014	2015
Rank of year x_i	3	4	5	6	7	8
VAT y_i (in millions LL)	600	700	750	950	1100	1350

- 1) Calculate the averages \bar{x} and \bar{y} of the two statistical variables x_i and y_i respectively.
- 2) Represent graphically the scatter plot as well as the center of gravity $G(\bar{x}; \bar{y})$ of the points $(x_i; y_i)$ in a rectangular system.
- 3) Write an equation of the regression line $D_{y/x}$ of y in terms of x and draw this line in the preceding system.
- 4) Suppose that the above pattern remains valid until the year 2020,
Estimate the VAT on cloths in the year 2020.

II- (4 points)

A shop sells products (perfumes, hair gel and shampoo) of two kinds A and B.

10% of kind A are “perfumes”, 30 % are “hair gel”, and the rest are “shampoo”

50% of kind B are “perfumes”, 20% are “hair gel”, and the rest “shampoo”

A client chooses one product at random.

Consider the events:

A: “The product is of kind A”

B: “The product is of kind B”

H: “The product is a hair gel”

F: “The product is a perfume”

S: “The product is a shampoo”

Suppose that $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{3}$.

1)

a- Calculate the following probabilities: $P(A \cap F)$, $P(A \cap H)$, $P(A \cap S)$, and $P(F)$.

b- Calculate the probability of the event: “The chosen product is of kind A, given that it is a perfume”

2) The prices of the products are given in the table below.

	Shampoo	Perfume	Hair Gel
A	LBP15 000	LBP80 000	LBP10 000
B	LBP10 000	LBP50 000	LBP5 000

Designate by X the random variable that is equal to the amount paid by the client.

- a- Determine the probability distribution of X .
- b- Calculate the mathematical expectation of X . Interpret the result.

III- (4 points)

In order to secure the future of their new-born, a bank proposes to parents the following offer:

For a deposit of 10 000 000 LL, an annual interest rate of 8 % is to be compounded annually, and to which a constant premium of 400 000 LL is to be added at the end of each year.

Designate by C_0 the initial capital ($C_0 = 10\,000\,000$), and by C_n the capital obtained at the end of the n th year.

1)

- a- Verify that $C_1 = 11\,200\,000$ and calculate C_2 . Deduce that the sequence (C_n) is neither arithmetic nor geometric.

- b- Express C_{n+1} in terms of C_n .

2) Consider the sequence (U_n) defined by: $U_n = C_n + 5\,000\,000$.

- a- Prove that (U_n) is a geometric sequence of common ratio 1.08 and whose first term is to be determined.

- b- Express U_n in terms of n . Deduce C_n in terms of n .

- c- How much shall be, after 18 years, the capital of a child whose parents accepted the offer of this bank?

IV-(8points)

The adjacent curve (C) is the representative of a continuous and strictly decreasing function h that is defined on $]0 ; +\infty[$ by:

$h(x) = a + bx - \ln(x)$ where a and b are two real numbers.

Indication: the line (d) of equation: $y = -1.2x + 4$ is tangent to the curve (C) at the point $(1; 2.8)$

A) 1. Prove that $a = 3$ and $b = -0.2$

2. Set up the table of variations of h .

B) Let g be the function defined over $[0 ; +\infty[$ by:

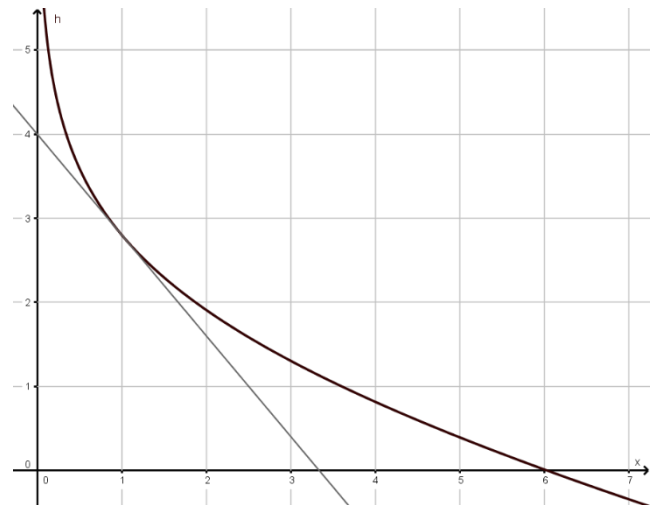
$g(x) = 3(1 - e^{-0.2x})$. Let (C_1) be the representative curve of g in an orthonormal system

1. Calculate $\lim_{x \rightarrow +\infty} g(x)$ and deduce an asymptote of (C) .
2. Study the variation of g and setup the table of variations.
3. (C_1) cuts (C) at a point of abscissa α . verify that $2.93 < \alpha < 2.95$
4. Draw (C_1) and (C) on the same curve.


C) In all what follows, let $\alpha = 2.94$

A factory produces a certain electronic articles. The demand, and the supply of this product in thousands of articles, are modeled by: $D(p) = 3(1 - e^{-0.2p})$ and $S(p) = 3 - 0.2p - \ln p$

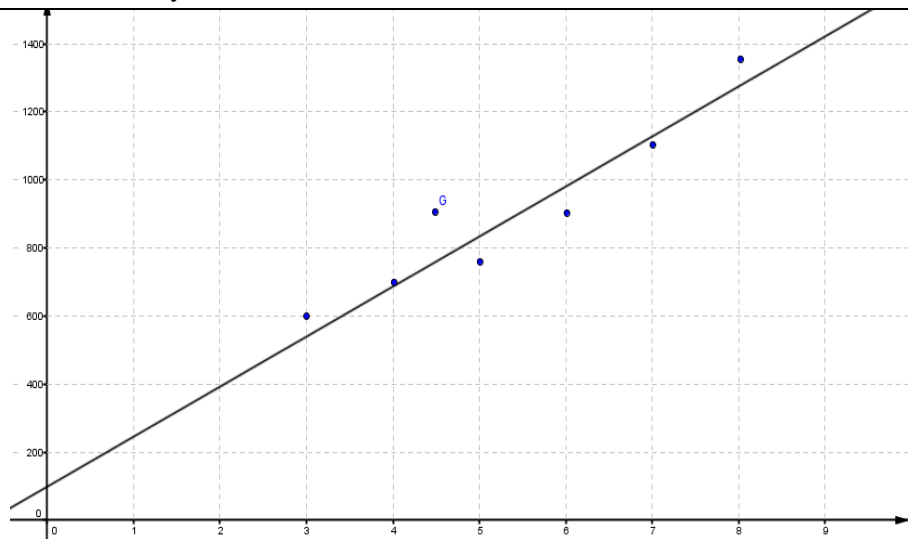
Where p is the unit price (price of one article) in thousands LL. ($0.2 \leq p \leq 5$).



1. Calculate the supply corresponding to a unit price of 2 000 LL.
2. Calculate the unit price for a demand of 4000 items.
3. Give an economical interpretation for the value 2.94 of α .
Calculate, in this case, the total revenue.
3. a- Determine $E(p)$, the elasticity of the demand with respect to the price p .
b- Calculate $E(2.94)$, and give an economical interpretation of the value thus obtained.

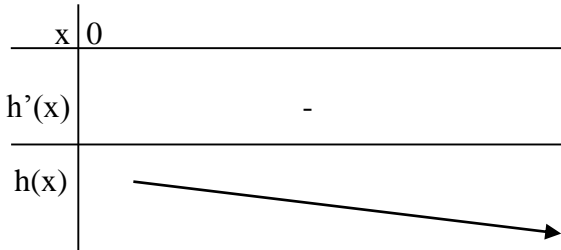
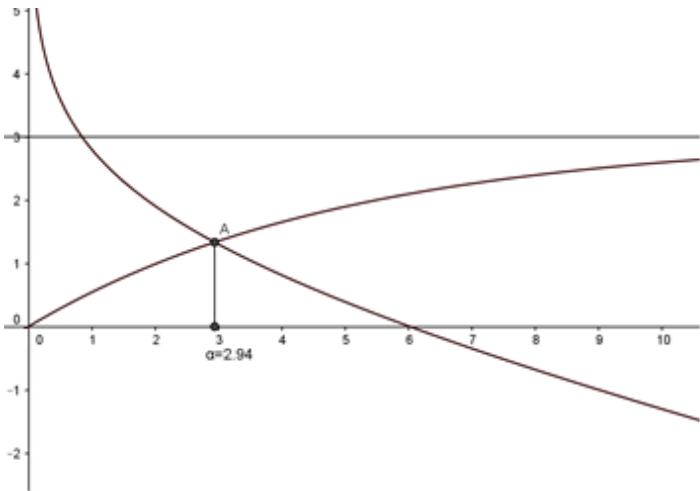
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أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي 2016-2017 وحتى صدور المناهج المطورة)

Question I		Mark
1	$\bar{x} = 5,5$ and $\bar{y} = 908,33$	1
2		1.5
3	$y = 147,142x + 99,047$	1.5
4	for $x = 13$ so $y = 147,142 \times 13 + 99,047 = 2011,893$ millions of LL	1

Question II										Mark
1)	a-	$P(A \cap F) = \frac{1}{15}, P(A \cap H) = \frac{1}{5}, P(A \cap S) = \frac{2}{15},$ $P(F) = P(A \cap F) + P(B \cap F) = \frac{2}{30} + \frac{5}{30} = \frac{7}{30}$								0.5 0.5 0.5 0.5
	b-	$P(F / A) = \frac{P(F \cap A)}{P(A)} = \frac{2}{7}$								0.5
2)	a-	$X = x_i$	5 000	10 000	15 000	50 000	80 000	Total		1
		$P(X = x_i)$	$\frac{1}{15}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{6}$	$\frac{1}{15}$	1		
	b-	$E(X) = \sum P_i \times x_i = 23$. The average amount paid by the client is 23 000 LL.								0.5
Question III										Mark
1)	a-	$C_1 = 10\,000\,000 + 10\,000\,000 \times 0.08 + 400\,000 = 11\,200\,000$ $C_2 = 11\,200\,000 + 11\,200\,000 \times 0.08 + 400\,000 = 12\,496\,000$ $\frac{C_1}{C_0} \neq \frac{C_2}{C_1}$ and $C_1 - C_0 \neq C_2 - C_1$								0.25 0.25 0.25 0.25
	b-	$C_{n+1} = C_n + 0.08C_n + 400\,000 = 1.08C_n + 400\,000$								0.5
2)	a-	$U_{n+1} = 1.08(C_n + 5\,000\,000) = 1.08U_n$; (U_n) is a geometric sequence of common ratio $r = 1.08$ and of first term $U_0 = 15\,000\,000$.								1
	b-	$U_n = U_0 \times r^n = 15 \times 10\,000\,000 \times 1.08^n$ and $C_n = 15 \times 10\,000\,000 \times 1.08^n - 5\,000$								0.5 0.5

c-	$C_{18} = 15\,000\,000 \times 1.08^{18} - 5\,000 = 54\,940\,000$; the capital of a child whose parents accepted the offer of this bank, after 18 years, is 54 940 000 LL	0.5
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Question IV		note									
A	<p>1) $h(1)=2,8$ donc $a+b=2,8$ $h'(1)=-1,2$ alors $b-1=-1,2$ alors $b=-0,2$ and $a=3$</p> <p>2)</p> 	1,5									
B-1	$\lim_{x \rightarrow +\infty} f(x) = 3$ $y=3$ asymptote horizontale.	0.5									
B-2	<p>$g'(x) = 0,6e^{-0,2x}$.</p> <table border="1" data-bbox="212 837 573 1039"> <tr> <td>x</td><td>0</td><td>$+\infty$</td></tr> <tr> <td>$g'(x)$</td><td></td><td>+</td></tr> <tr> <td>$g(x)$</td><td>0</td><td>3</td></tr> </table>	x	0	$+\infty$	$g'(x)$		+	$g(x)$	0	3	1
x	0	$+\infty$									
$g'(x)$		+									
$g(x)$	0	3									
B-3	<p>Let $L(x)=f(x)-g(x)$ $L(2,93) \times L(2,95) < 0$ Donc $2,93 < \alpha < 2,95$</p>	0,5									
B-4		1									
C-1	$S(2)=1.906$ thousands articles	0.5									
C2	$D(p)=2$ so $e^{-0,2p}=\frac{-1}{3}$ then $p=\ln(3)/0.2$ donc $p=5.4930$ thousands LL.	0.5									
C-3	<p>2.94 is the price of equilibrium $R=2.94 \times D(2.94)=3.921$ millions LL.</p>	1									

C-4-a	$e(p) = \frac{0.2p e^{-0.2p}}{1 - e^{-0.2p}}$	0.5
C-4-b	$e(2.94) = 0.73$ if the price increase 1% from 2.94 in thousands of LL the demand decrease 0,73%	1