المادة: الرياضيات الشهادة: المتوسطة نموذج رقم -١- المدّة : ساعتان	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المركز التربوي للبحوث والإنماء

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I - (2 points)

Consider the three numbers A, B and C:

A= $\frac{33 \text{ x } 10^{-4} \text{x } 30 \text{ x } 10^{2}}{36 \text{ x } 10^{-2} \text{x } 22 \text{ x } 10}$; B= $\frac{7-\frac{11}{3}}{1-\frac{1}{6}}$; C= $(\sqrt{2}-1)^2+(\sqrt{2}+1)^2$

All details of calculation must be shown.

1) Write A as a fraction in its simplest form.

2) Show that B is a natural number.

3) Verify that C = B+16A.

II – (3 points)

The perimeter of a rectangle is 28cm. If the length is decreased by 10% and the width is increased by 20%, then the perimeter of this rectangle will be 28.8cm.

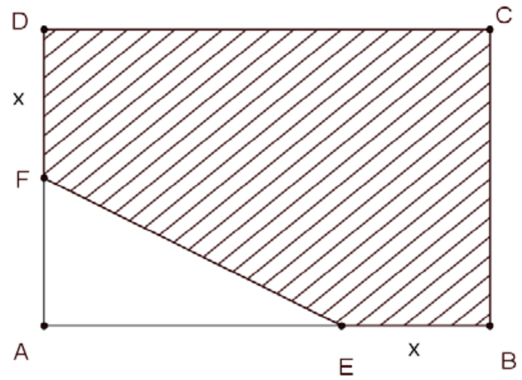
a) Write a system of 2 equations of 2 unknowns to model the previous text.

b) Verify that the original length is 8cm and calculate the original width.

c) Determine the nature of quadrilateral resulting from modification of dimensions of the rectangle.

III – (4 points) in the figure at the right :

- x *is a length expressed in cm such that* $0 < x < 4$.
- ABCD is a rectangle such that AB=6cm and AD= 4cm.
- BE = DF = x



Denote by Y the area of the shaded part.

1) Prove that Y = $-\frac{1}{2}(x^2-10x-24)$

2) a. Verify that Y= $-\frac{1}{2}((x-5)^2-49)$.

b. Determine x so that y = 20.

3) Z is the area of a square with side (x+2).

a. Express Z in terms of x.

b. Simplify $\frac{Y}{Z}$.

c. Can we calculate x if Y = Z ?

IV - (5.5 points)

In an orthonormal system of axes (x'Ox, y'Oy), consider the points A(3; 0) and B(-1; 2).

Let (d) be the line with equation y = 2x + 4.

1) a. Plot the points A and B.

b. The line (d) intersects x'Ox at E and y'Oy at F. Calculate the coordinates of points E and F, then draw (d).

c. Verify that B is the midpoint of [EF].

2) a. Determine the equation of line (AB).

b. Verify that (AB) is perpendicular bisector of [EF].

3) Consider the point H(0 ; $\frac{3}{2}$).

a. Verify, that H is on the line (AB).

b. Show that H is the orthocenter of the triangle AEF.

4) Let (C) be the circle with diameter [AF] and (Δ) the line passing through A and parallel to (EH).

a. Verify that O and B are on the circle (C).

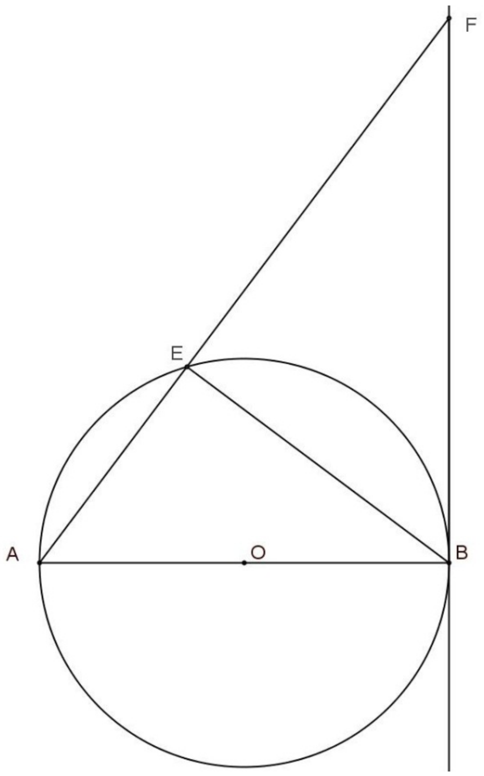
b. Write an equation of the line (Δ).

c. Show that (Δ) is the tangent to (C).

V- (5.5 points)

In the adjacent figure at rhe right:

- AB =5 cm.
- (C) is the circle with diameter [AB] and center O.
- E a point on (C) such that AE = 3cm.
- The tangent to (C) at B intersect (AE) at F.



1) Copy the figure.

2) **a.** Calculate BE

b. Prove that the two triangles AEB and ABF are similar.

c. Deduce BF and EF.

3) L is a point on (FB) such that BL= $\frac{15}{4}$**, B is between L and F.**

a. Compare $\frac{FE}{EA}$ and $\frac{FB}{BL}$.

b. Deduce that (BE) is parallel to (AL).

c. Show that AL = $\frac{25}{4}$

4) The line (EO) intersects the circle (C) at H. Let G the midpoint of[BL].

a. Prove that EAHB is a rectangle. Deduce that H is on (AL).

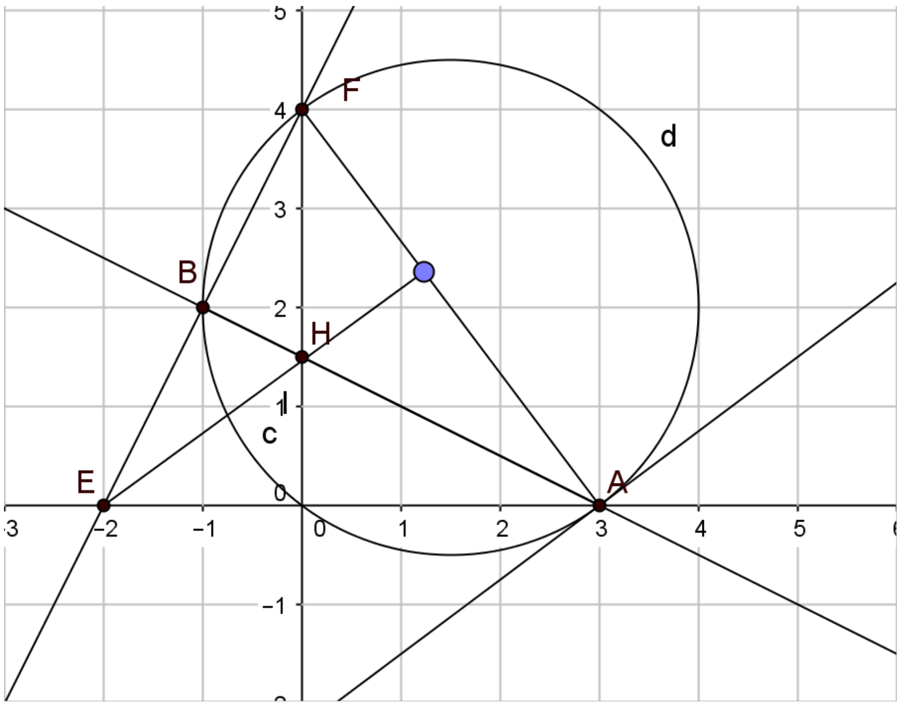
b. Prove that (GH) is tangent to (C).

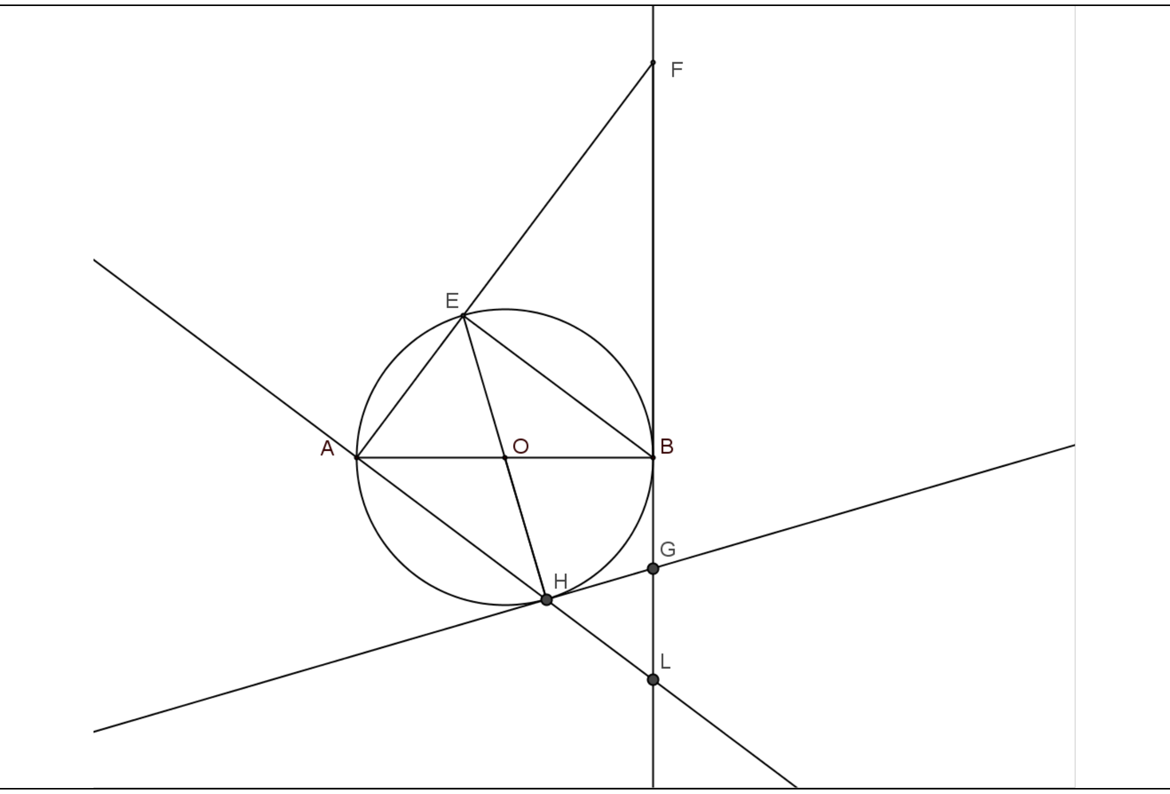
c. Calculate, rounded to the nearest degree, the measure of $\widehat{GHB}$.

المركز التربوي للبحوث والإنماء	الهيئة الأكاديميّة المشتركة قسم : الرياضيات	المادة: الرياضيات الشهادة: المتوسطة نموذج رقم -١- المدّة : ساعتان

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

	Question I	
	Answers	**note**
1	$A = \frac{33 \text{ x } 10^{-4} \text{x } 30 \text{ x } 10^{2}}{36 \text{ x } 10^{-2} \text{x } 22 \text{ x } 10} = \frac{9\text{x } 10^{-1}}{72\text{x } 10^{-1}} = \frac{1}{8}$ ¼ + ¼ $B = \frac{\frac{10}{3}}{\frac{5}{6}} = 4$, ¼ + ¼ C $=(\sqrt{2}-1)^2+(\sqrt{2}+1)^2 = 2-2\sqrt{2}+1+2+2\sqrt{2}+1=6$ ¼ + ¼ +¼	**1¾**
2	$16A + B = 2 + 4 = 30$ $C = 6$, so C = B +16A.	¼
	Question II	
a	2x +2y=28cm (1-0,1)x+(1+0.2)y=28cm	**1¼**
b	x=8, y= 6	**1**
c	1,2y=7.2 et 0,9x =7.2 Therefore the quadrilateral is a square.	¾
	Question III	
1	Area of hatched area Y = $24 - \frac{(4-x)(6-x)}{2} = \frac{-x^2+10x+24}{2} = -\frac{1}{2}(x^2-10x-24)$.	**1**
2.b	20= $-\frac{1}{2}((x-5)^2 - 49)$ alors$(x-5)^2 - 49 = -40$, $(x-5)^2 = 9$ x-5=3 or x-5=-3 so x=8(unacceptable) ou x=2. ¼ + ¼ +¼ + ¼ +¼	1¼
3.a	Z = $(x+2)^2$	¼
3.b	$\frac{Y}{Z} = \frac{-\frac{1}{2}(x-12)(x+2)}{(x+2)^2} = \frac{-\frac{1}{2}(x-12)}{(x+2)} = \frac{-(x-12)}{2(x+2)}$ (with $x \neq -2$)	½
3.c	Y= Z so $\frac{-(x-12)}{2(x+2)}$ =1 then $-(x-12) = 2(x+2)$ so x= $\frac{8}{3}$ acceptable.	**1**

Question IV		
1.a		½
1.b	E(0;-2) and F(0 ; 4)	½
1.c	$x_B = \frac{(xE+xF)}{2}$ $y_B = \frac{(yE+yF)}{2}$	½
2.a	The equation of (AB) : y = a x+b $a(AB) = \frac{(yB-yA)}{(xB-xA)} = \frac{-1}{2}$ and $y_B = \frac{-1}{2} x_B + b$ so $b = \frac{3}{2}$.	¾
2.b	slope (AB) × slope (d) = -1 and (AB) through B middle of [EF] so (AB) is the mediator of [EF].	½
3.a	$y_H = \frac{-1}{2} x_H + \frac{3}{2}$. so H is a point of (AB)	¼
3.b	(FH) ⊥ at (EA) and (AB) ⊥ at (EF) , (AB) and (FH) meet in H then H is the orthocenter of the triangle AEF.	¾
4.a	$\widehat{ABF} = 90°$ (ABF triangle inscribed in a semicircle of diameter [AF]) $\widehat{AOF} = 90°$ (AOF triangle inscribed in a semicircle of diameter [AF]) Therefore B and O are two points of the circle.	½
4.b	The equation of (Δ) : y = a x+b $a(\Delta) = a(EH) = \frac{(yE-yH)}{(xE-xH)} = \frac{3}{4}$ and $y_A = \frac{3}{4} x_A + b$ so $b = \frac{9}{4}$.	¾
4.c	(EH) ⊥ at (FA) and (Δ)//at (EH) then (Δ)⊥ at (FA) in A so (Δ) is tangential to the circle (C) in A.	½

	Question V	
1		½
2.a	In the triangle AEB rectangle in E. According to Pythagoras $BE^2 = AB^2 - AE^2$, BE = 4.	½
2.b	The two triangles BDE and BAD are similar because: $\hat{A}$ common angle $\widehat{AEB} = \widehat{ABF} = 90$	½
2.c	Similarity ratio: $\frac{AE}{AB} = \frac{AB}{AF} = \frac{EB}{BF}$ ¼ $\frac{3}{5} = \frac{5}{AF} = \frac{4}{BF}$ so BF= $\frac{20}{3}$ and AF= $\frac{25}{3}$ so EF= $\frac{25}{3}$ −3= $\frac{16}{3}$.	½
3.a	$\frac{EF}{EA} = \frac{16}{9}$ so $\frac{FB}{BL} = \frac{16}{9}$.	½
3.b	$\frac{EF}{EA} = \frac{FB}{BL}$, then the two straight lines (EB) and (AL) are parallel according to the reciprocal of Thales.	½
3.c	$\frac{EF}{FA} = \frac{EB}{AL}$ so AL= $\frac{15}{4}$.	½
4.a	The two triangles HBL and BAH are similar because $\widehat{BAH} = \widehat{HBL} = \frac{\widehat{AB}}{2}$	**1**

	$\frac{AH}{HB} = \frac{4}{3}$ and $\frac{AB}{BL} = \frac{4}{3}$ so $\frac{AH}{HB} = \frac{AB}{BL}$ And consequently $\widehat{BHL} = \widehat{ABL}$ = 90 then $\widehat{BHL}$+$\widehat{BHA}$ =180 so H is on (AL).	
5.b	In the triangle BHL rectangle in H on HG = GB = GL (the median is half the hypotenuse) Then the two triangles OBG and OHG are isometric. $\widehat{GHO} = \widehat{OBL} = 90$ then BH tangent to (C).	**½**
5.c	cos $\widehat{GBH} = \frac{BH}{BL} = \frac{3}{\frac{15}{4}} = \frac{4}{5}$ Alors $\widehat{GBH} = cos^{-1}\left(\frac{4}{5}\right)$=36,8°≈ 37°	**½**