

المادة: الرياضيات الشهادة: الثانوية العامة الفرع: العلوم العامة نموذج رقم -١- المدة: أربع ساعات	الهيئة الأكاديمية المشتركة قسم: الرياضيات	 المركز التربوي للبحوث والإنماء
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نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

- ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I- (2points)

Answer true or false and justify.

- 1) The complex number $(-1+i)^{10}$ is real .
- 2) The function f' defined as $f'(x) = \int_0^{x^2} \sqrt{t^2 + 4} dt$ is the derivative function of a function f . then the function f doesn't have an inflection point over $\mathbb{R}$.
- 3) If $f(x) = x^2 e^x$, then the n th derivative is $f^{(n)}(x) = (x^2 + 2nx + (n-1))e^x$ for all $n \in \mathbb{N}^*$.
- 4) $1+i + \dots + i^{19} = 0$ (i is imaginary).

II- (2points).

In the plane referred to an orthonormal system $(O ; i , j , k)$, consider the plane (P) with equation $3x+y-5=0$, and the lines (D) and (D') defined as :

$$(D) : \begin{cases} x = t \\ y = -3t + 5 \\ z = t - 4 \end{cases} \quad (D') : \begin{cases} x = m + 1 \\ y = -2m - 1 \\ z = -m + 3 \end{cases}$$

- 1)
 - a) Verify that (P) is perpendicular to the plane (XOY) .
 - b) Prove that (D) is included in (P) .
- 2) Prove that (D) and (D') intersect at a point A with coordinates to be determined .
In what follows , given the point $B(0,1,4)$.
- 3) Consider in the plane (Q) formed by (D) and (D') the circle (C) with center A and radius AB .
 - a) Write an equation of (Q) .
 - b) Write a system of parametric equations of the line (Δ) , tangent at B to the circle (C) .
- 4) Calculate the coordinates of E and F , the intersection points of the circle (C) with the line (D) .

III- (3points)

Consider two boxes U and V .

- The box U contains 10 cards where three of them have the letter A, 5 have the letter B and 2 the letter C.
- The box V contains 6 balls where two of them are red and four are black .

The rule of game is the following :

One card is randomly selected from the box U.

- If the player selects a card A then he draws two balls from the box V , one after another with replacement
- If the player selects a card B, then he draws two balls from the box V , one after another without replacement .
- The game stops when the player selects a card C or he selects one black ball .

Consider the following events :

A : « Select a card named A »

B : « Select a card named B »

C : « Select a card named C »

G : « The player wins »

The player wins only if he selects 2 red balls one after another or he selects a card C .

- 1) Calculate $P(G/A)$ and show that $P(G \cap A) = \frac{1}{30}$.
- 2) Calculate $P(G \cap B)$, then $P(G)$.
- 3) To participate in this game , the player should pay 2000 LL. He wins 5000 LL if he selects a card named C and 3000 LL if he selects 2 red balls .

Let X be the random variable that is equal to the amount got by the player .

- a) Show that the three values of X are -2000, 1000 and 3000.
- b) Determine the probability distribution of X .
- c) Estimate the amount got by the organizer if 100 players participate to this game.

IV- (3 pts)

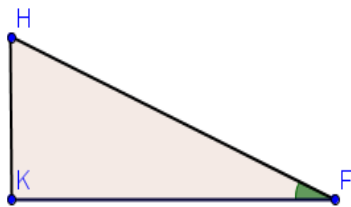
In the oriented plane , given a rectangle ABCD with center O so that $AB = 4\text{cm}$,

and $(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{\pi}{6} (2\pi)$.

Let E be the symmetric of A with respect to D . Denote by S the similitude that maps E onto O and A onto B . .

1. Verify that the ratio of the similitude is $k = \frac{1}{2}$ and determine a measure of the angle α of S.
2. Determine the image of D under S . Show that C is the center of S .
3. Let I be the point of $[EO]$, distinct from E and O ; and (Γ) the circle with center I and passing through A . (Γ) intersects (AD) and (AB) respectively at M and P.
 - a. Draw (Γ) and plot the points M and P .
 - b. Justify that $C \in (\Gamma)$.
4. Let N be the orthogonal projection of C on (MP).
 - a. Show that $(\overrightarrow{MP}, \overrightarrow{MC}) = \frac{\pi}{6} (2\pi)$.
 - b. Deduce that $S(M) = N$.
5. Prove that B , N and D are colinear .
6. The plane is referred to an orthonormal direct system $(A, \vec{u}, \vec{v})$, with $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.
 - a. Determine the affixes of the points B and C.
 - b. Give the complex form of S.

V-(3points)



In the next figure , FKH is a right triangle at K with $FK=3\text{cm}$ and $KH=\sqrt{3}\text{ cm}$.

Let A be a point on [FK] so that $AK=1\text{cm}$ and A' is the symmetric of F

With respect to K .

Denote by (H) the hyperbola with focus F, and directrix (KH) and eccentricity 2 .

- 1.a Determine the focal axis of (H) .
 - b. Prove that A and A' are the vertices of (H) .
- 2.a. Determine the center O of (H) and the second focus F'.

b. Show that (OH) is an asymptote of (H) then find the second asymptote .

c. Draw (H).

3. Let G be a point so that $\overrightarrow{FG} = 2\sqrt{3}\overrightarrow{KH}$, prove that G is a point on (H).

4. The plane is referred to an orthonormal system $(O, \vec{u}, \vec{v})$, with $\vec{u} = \overrightarrow{OK}$.

a. a- Verify that an equation of (H) is : $\frac{x^2}{4} - \frac{y^2}{12} = 1$

b. b- Prove that (GK) is tangent to (H).

VI-. (7pts)

The plane is referred to the orthonormal system $(O, \vec{i}, \vec{j})$.

(Part A)

Consider the differential equation (E) defined as : $y' + y = 1 - 2e^{-x+1}$.

1. Determine a and b so that $Y = a + bxe^{-x}$ is a particular solution of (E).

2. Solve (E). Deduce the particular solution of (E) so that $y(1) = 0$.

(Part B)

Let g be a function defined over $[0; +\infty[$, as $g(x) = 1 + (1 - 2x)e^{-x+1}$. (C) is its representative curve .

1. Determine $\lim_{x \rightarrow +\infty} g(x)$. Give a geometric interpretation.

2. Calculate $g'(x)$ the derivative of $g(x)$. Set up the table of variations of g .

3. Prove that $g(x) = 0$ has two roots 1 and α so that $\alpha \in [2.25, 2.26]$.

Verify that $e^{\alpha-1} = 2\alpha - 1$.

4. Solve $g(x) \leq 0$. Deduce the solutions of the inequality $g(x^2) \leq 0$.

5. Draw (C) .

6. Calculate the area of the region bounded by (C), the line (Δ) with equation $y = 1$ and the two lines with equations $x = 1$ and $x = 2$.

(Part C)

Let f be the function f defined over $[0, +\infty[$ as $f(x) = 1 + x + xe^{-x^2+1}$. (Γ) its representative curve

and (d) the line with equation $y = x + 1$.

1. Calculate $f(-x) + f(x)$. Make a conclusion .

2. Determine $\lim_{x \rightarrow +\infty} f(x)$. Show that (d) is an asymptote when x tends to $+\infty$.

Discuss the relative position of (Γ) and (d) .

3. Prove that $f'(x) = g(x^2)$ for all $x \in [0, +\infty[$. Verify that $f(\sqrt{\alpha}) = 1 + \frac{2\alpha\sqrt{\alpha}}{2\alpha - 1}$.

4. Set up the table of variations of f .

5. Draw (Γ) .

6. Let $(U_n)_{n \geq 0}$ be the sequence defined as : $U_n = \int_0^1 [f(nx) - nx] dx$.

a. Calculate U_0 .

b. Write U_n in terms of n . Determine $\lim_{n \rightarrow +\infty} U_n$.

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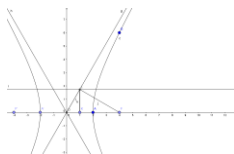
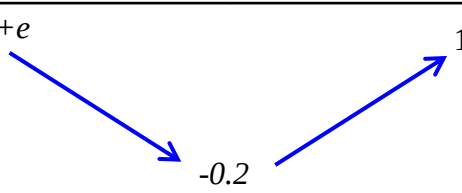
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

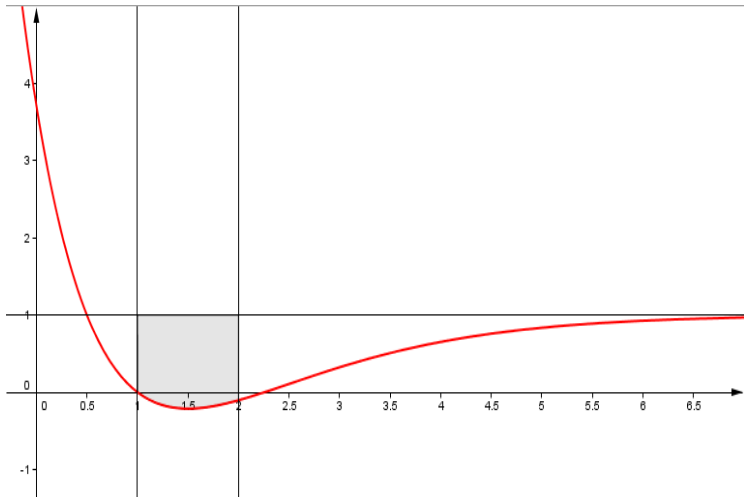
QI		Notes
1	False : on the y – axis .	1
2	False :its has at x=0 an inflection point .	1
3	False : for n=2	1
4	True : its a sum of a geometric sequence with first term 1 and ratio q=i	1

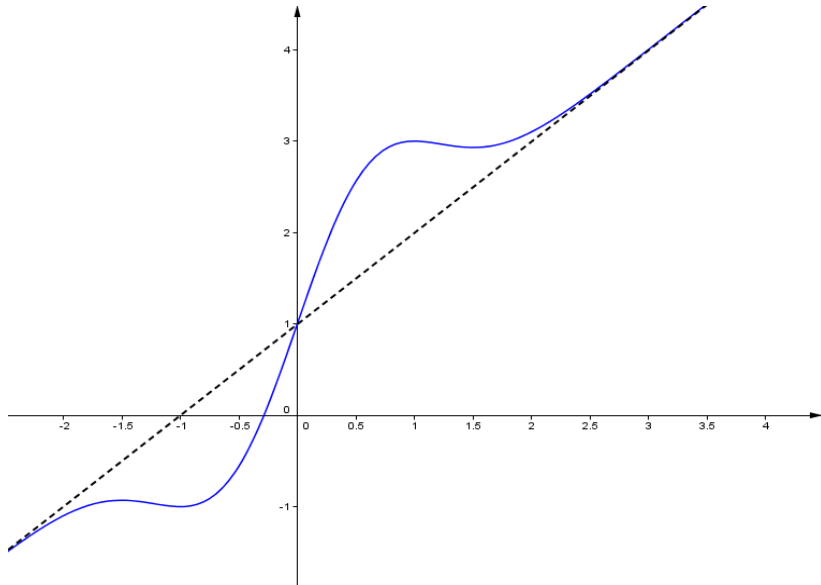
QII		Notes
1.a	$(3,1,0)(0,0,1) = 0$ Then (P) is perpendicular to the plane XOY	0,5
1.b	$3t-3t+5-5=0 \Rightarrow (D) \subset (P)$	0,5
2	A(4,7,0) for m=3 et t=4	0,5
3.a	(Q) : $5x+2y+z-6=0$	0,5
3.b	$\vec{u} = \vec{AB} \wedge \vec{n}$ with $\vec{u}$ direction vector of the tangent and $\vec{n}$ normal vector of (Q). $(\Delta) : \begin{cases} x = 0 \\ y = 6\lambda + 1 \\ z = -12\lambda + 4 \end{cases}$	1
4	$t = 4 + \frac{4\sqrt{66}}{11}$ et $t = 4 - \frac{4\sqrt{66}}{11}$	1

QIII						Notes
1	$P(G / A) = \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$. $P(G \cap A) = P(G / A) \times P(A) = \frac{1}{9} \times \frac{3}{10} = \frac{1}{30}$					1
2	$P(G \cap B) = P(G / B) \times P(B) = \frac{2}{6} \times \frac{1}{5} \times \frac{1}{2} = \frac{1}{30}$					1
	$P(G) = P(G \cap A) + P(G \cap B) + P(C) = \frac{4}{15}$					1
3.a	$-2000(\overline{G})$, $1000(G \cap A \text{ ou } G \cap B)$, $3000(C)$.					1
3.b	x_i	-2000	1000	3000		1
	$P(X=x_i)$	$\frac{11}{15}$	$\frac{1}{15}$	$\frac{1}{5}$		
3.c	$E(x) = -800$ Then the gain of the organizer is $800 \times 100 = 80\,000$ LL.					1

Q4		
1.	$k = \frac{OB}{EA}$, using the equilateral triangle OBC: $k = \frac{BC}{2BC}$	
	$k = \frac{1}{2}$ et $\alpha = (\overrightarrow{AE}, \overrightarrow{BO}) = (BC, BO) = \frac{\pi}{3} \quad (2\pi)$	1

2.b	The tangent of the angle formed by (OH) and the focal axis is equal to $\sqrt{3} = \frac{b}{a}$ with $a=OA=2$ $C=OF=4$ et $c^2 = a^2 + b^2$ The second asymptote is symetric of (OH) with respect the perpendicular line at O.	1 0.5												
2.c		0.5												
3.	$\frac{GF}{d(G/(HK))} = 2$ then G is on (H).	0.5												
4.a	Since $a=2$ and $b = 2\sqrt{3}$,centre O and the focal axe is $x'ox$.	0.5												
4.b	G(4 ;6) and K(1,0) and (GK) : $y=2x-2$ is the tangent at G to (H).	1												
Q.6														
part A														
1	$Y=a + bxe^{-x}$, Y verify (E), then $a =1$ and $b= -2e$	.0.5												
.2	The general solution is : $y=ce^{-x} + Y = Ce^{-x} +1 - 2xe^{-x+1}$ $y(1)= 0$ then $C = e$. Particular solution : $y = 1 + (1-2x)e^{-x+1}$	1												
Part B														
1	$\lim_{x \rightarrow -\infty} g(x) = 1$ then $y = 1$ is an horizontal asymptote .	0.5												
2	$g'(x) = (2x - 3)e^{-x+1}$ <table border="1" data-bbox="503 1543 1148 1892"><tr><td>x</td><td>0</td><td>$\frac{3}{2}$</td><td>$+\infty$</td></tr><tr><td>$g'(x)$</td><td>—</td><td>0</td><td>+</td></tr><tr><td>$g(x)$</td><td>$1+e$</td><td></td><td>1</td></tr></table> 	x	0	$\frac{3}{2}$	$+\infty$	$g'(x)$	—	0	+	$g(x)$	$1+e$		1	1.5
x	0	$\frac{3}{2}$	$+\infty$											
$g'(x)$	—	0	+											
$g(x)$	$1+e$		1											
3	$g(1) = 0$ then 1 is a root . $g(2.25) = -2.77 \bullet 10^{-3}$	1												

	$g(2.26) = 1.54 \cdot 10^{-3}$ g is continuous and strictly decreasing on $[2, 25 ; 2, 26]$ then α is a root. $g(\alpha) = 0$ if and only if $e^{\alpha-1} = 2\alpha - 1$.	
4	$g(\alpha) \leq 0$ if and only if $x \in [1, \alpha]$. $g(x^2) \leq 0$ if and only if $x^2 \in [1, \alpha]$, if and only if $x \in [1, \alpha]$.	1
5		1.5
6	$A = \int_1^2 [y_{(\Delta)} - g(x)] dx = \int_1^2 [-(1-2x)e^{-x+1}] dx$ With integration by part : $\int_1^2 -(1-2x)e^{-x+1} dx = 3 - \frac{5}{e} \approx 1.1606$ $A = 1.16u^2$.	1
partie C		
1	$f(-x) + f(x) = 2$, and f is centered at 0. $I(0,1)$ is center of symmetry of the curve.	0.5
2	$\lim_{x \rightarrow +\infty} f(x) = +\infty$ et $\lim_{x \rightarrow +\infty} [f(x) - y_{(d)}] = 0$. $f(x) - y_{(d)} = x e^{-x^2+1}$. $x e^{-x^2+1} > 0$ if $x > 0$ then (Γ) is above (d) . $x e^{-x^2+1} = 0$ if $x = 0$ then (Γ) intersects (d) . $x e^{-x^2+1} < 0$ then (Γ) is below (d) .	1
3	$f'(x) = g(x^2)$ $f(\sqrt{\alpha}) = 1 + \frac{2\alpha\sqrt{\alpha}}{2\alpha-1}$.	0.5

4	<table> <tr> <td>x</td> <td>0</td> <td>1</td> <td>$\sqrt{\alpha}$</td> <td>$+\infty$</td> </tr> <tr> <td>$f'(x)$</td> <td>+</td> <td>0</td> <td>-</td> <td>0</td> <td>+</td> </tr> <tr> <td>$f(x)$</td> <td>1</td> <td>3</td> <td>$f(\sqrt{\alpha})$</td> <td>$+\infty$</td> </tr> </table>	x	0	1	$\sqrt{\alpha}$	$+\infty$	$f'(x)$	+	0	-	0	+	$f(x)$	1	3	$f(\sqrt{\alpha})$	$+\infty$	1
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6.a	$U_0 = \int_0^1 1 dx = 1.$	0.5																
6.b	$U_n = \int_0^1 (1 + nxe^{-(nx)^2+1}) dx = 1 + \int_0^1 (nxe^{-(nx)^2+1}) dx$ Let $v = -(nx)^2 + 1$, then $dv = -2n^2x dx$, $nxdx = -\frac{dv}{2n}$. $U_n = 1 + \int_1^{-n^2+1} -e^v \frac{dv}{2n} = 1 - \frac{1}{2n} (e^{-n^2+1} - e).$ $\lim_{n \rightarrow +\infty} U_n = 1$	1																