| المركز التربوي للبحوث والإنماء | الهيئة الأكاديميّة المشتركة<br>قسم : الرياضيات | المادة: الرياضيات<br>الشهادة: الثانوية العامة<br>الفرع: الاجتماع والاقتصاد<br>نموذج رقم -١-<br>المدّة : ساعتان |
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**نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)**

ارشادات عامة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

**I- (4 points)**

The table below shows the VATon clothsy$_i$, in the last 6 years in a certain country

| Year | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|---|
| Rank of year $x_i$ | 3 | 4 | 5 | 6 | 7 | 8 |
| VAT$y_i$ (in millions LL) | 600 | 700 | 750 | 950 | 1100 | 1350 |

1) Calculate the averages $\overline{x}$ and $\overline{y}$ of the two statistical variables $x_i$ and $y_i$ respectively.
2) Represent graphically the scatter plot as well as the center of gravity G ($\overline{x}$ ; $\overline{y}$) of the points ($x_i$ ; $y_i$) in a rectangular system.
3) Write an equation of the regression line $D_{y/x}$ of y in terms of *x* and draw this line in the preceding system.
4) Suppose that the above pattern remains valid until the year 2020, Estimate the VAT on cloths in the year 2020.

**II- (4 points)**

A shop sells products (perfumes, hair gel and shampoo) of two kinds A and B.
10% of kind A are “perfumes”, 30 % are “hair gel”, and the rest are “shampoo”
50% of kind B are “perfumes”, 20% are “hair gel”, and the rest “shampoo”
A client chooses one product at random.
Consider the events:
**A**: “The product is of kind A”
**B**: “The product is of kind B”
**H**: “The product is a hair gel”
**F**: “The product is a perfume”
**S**: “The product is a shampoo”

Suppose that $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{3}$.

1)
a- Calculate the following probabilities: $P(A \cap F)$, $P(A \cap H)$, $P(A \cap S)$, and P(F).
b- Calculate the probability of the event: “The chosen product is of kind A, given that it is a perfume”
2) The prices of the products are given in the table below.

| | Shampoo | Perfume | Hair Gel |
|---|---|---|---|
| A | LBP15 000 | LBP80 000 | LBP10 000 |
| B | LBP10 000 | LBP50 000 | LBP5 000 |

Designate by X the random variable that is equal to the amount paid by the client.

a- Determine the probability distribution of X.

b- Calculate the mathematical expectation of X. Interpret the result.

**III- (4 points)**

In order to secure the future of their new-born, a bank proposes to parents the following offer:
For a deposit of 10 000 000 LL, an annual interest rate of 8 % is to be compounded annually, and to which a constant premium of 400 000 LL is to be added at the end of each year.
Designate by $C_0$ the initial capital ($C_0 = 10\ 000\ 000$), and by $C_n$ the capital obtained at the end of the nth year.

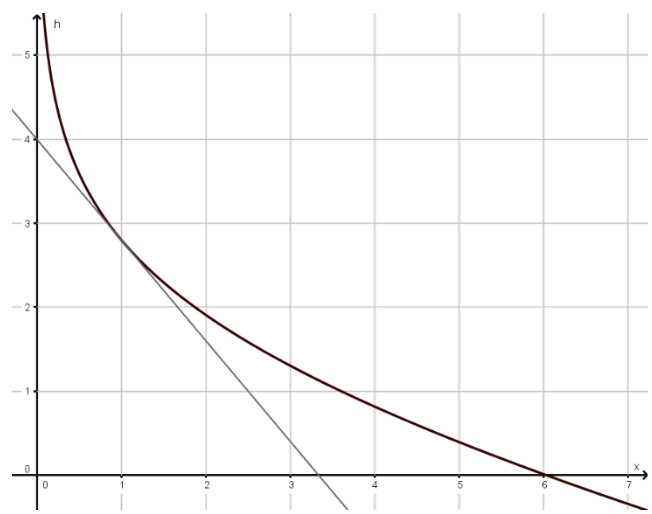
1)
   a) Verify that $C_1 = 11\ 200\ 000$ and calculate $C_2$. Deduce that the sequence $(C_n)$ is neither arithmetic nor geometric.
   b) Express $C_{n+1}$ in terms of $C_n$.
2) Consider the sequence $(U_n)$ defined by: $U_n = C_n + 5\ 000\ 000$.
   a) Prove that $(U_n)$ is a geometric sequence of common ratio 1.08 and whose first term is to be determined.
   b) Express $U_n$ in terms of n. Deduce $C_n$ in terms of n.
   c) How much shall be, after 18 years, the capital of a child whose parents accepted the offer of this bank?

**IV-(8points)**

The adjacentcurve (C) is the representative of a continuous and strictly decreasing function h that is defined on $]0 ; +\infty[$by:

$h(x) = a + bx - \ln(x)$ where a and b are two real numbers.

**Indication:** the line (d) of equation: $y = -1.2x + 4$ is tangent to the curve (C) at the point (1; 2.8)




**A)**

1) Prove that a = 3 and b = –0.2
2) Set up the table of variations of h.

**B)**

Let g be the function defined over $[0 ; +\infty[$ by:
$g(x) = 3(1-e^{-0\text{-}2x})$.Let $(C_1)$ be the representative curve of g in an orthonormal system

1) Calculate $\lim_{x\to+\infty} g(x)$ and deduce an asymptote of (C).
2) Study the variation of g and setup the table of variations.
3) $(C_1)$ cuts (C) at a point of abscissa α. verify that $2.93 < \alpha < 2.95$
4) Draw $(C_1)$ and (C) on the same curve.

**C)**

In all what follows, let α = 2.94

A factory produces a certain electronic articles. The demand, and the supply of this product in thousands of articles, are modeled by: $D(p) = 3(1 - e^{-0.2p})$ and $S(p) = 3 - 0.2p - \ln p$
Where p is the unit price (price of one article) in thousands LL. $(0.2 \leq p \leq 5)$.

1) Calculate the supply corresponding to a unit price of 2 000 LL.
2) Calculate the unit price for a demand of 4000 items.
3) Give an economical interpretation for the value 2.94 of $\alpha$.
   Calculate, in this case, the total revenue.
   a) Determine E(p), the elasticity of the demand with respect to the price p.
   b) Calculate E(2.94), and give an economical interpretation of the value thus obtained.

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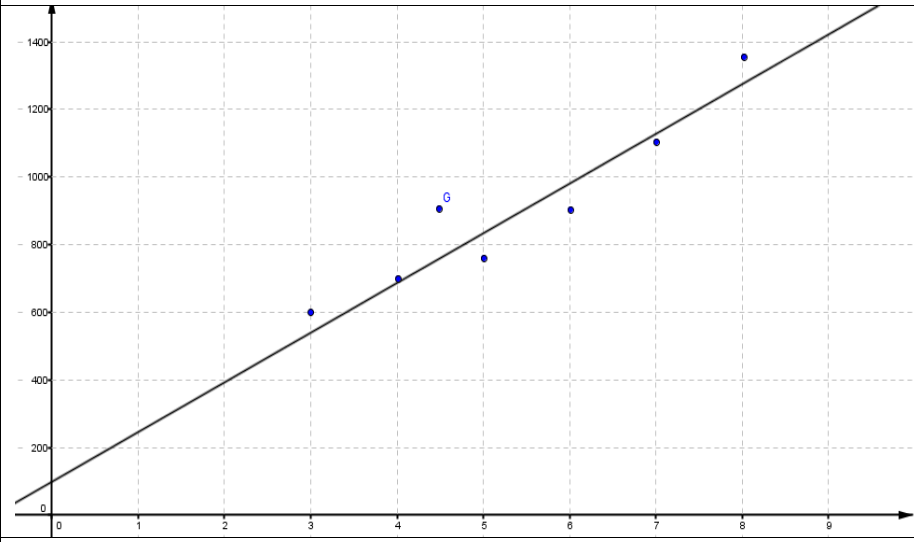
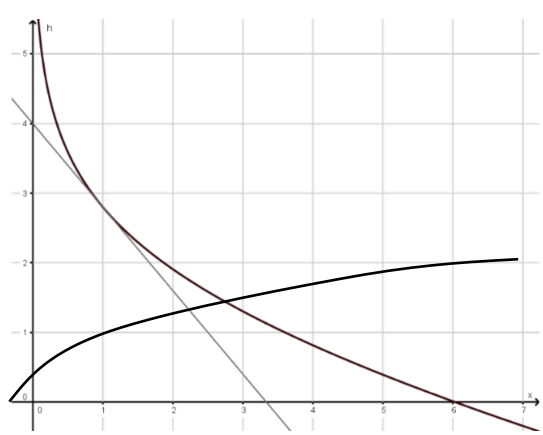
أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدّل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطوّرة)

| | **Question I** | **Mark** |
|---|---|---|
| 1 | $\bar{x} = 5{,}5$ and $\bar{y} = 908{,}33$ | 1 |
| 2 |  | 1.5 |
| 3 | $y = 147{,}142x + 99{,}047$ | 1.5 |
| 4 | for $x = 13$ so $y = 147{,}142 \times 13 + 99{,}047 = 2011{,}893$ millions of LL | 1 |

| | | **Question II** | **Mark** |
|---|---|---|---|
| 1) | a- | $P(A \cap F) = \frac{2}{5},\ P(A \cap H) = \frac{1}{5},\ P(A \cap S) = \frac{2}{5},$<br>$P(F) = P(A \cap F) + P(B \cap F) = \frac{2}{5} + \frac{5}{30} = \frac{17}{30}$ | **0.5**<br>**0.5**<br>**0.5**<br>**0.5** |
| | b- | $P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{12}{17}$ | **0.5** |
| 2) | a- | $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|} \hline X = x_i & 5\,000 & 10\,000 & 15\,000 & 50\,000 & 80\,000 & \text{Total} \\ \hline P(X = x_i) & \frac{1}{15} & \frac{3}{10} & \frac{2}{5} & \frac{1}{6} & \frac{1}{15} & 1 \\ \hline \end{array}$ | **1** |
| | b- | $E(X) = \sum P_i \times x_i = 23$. The average amount paid by the client is 23 000 LL. | **0.5** |

| | | **Question III** | **Mark** |
|---|---|---|---|
| 1) | a- | $C_1 = 10\,000\,000 + 10\,000\,000 \times 0.08 + 400\,000 = 11\,200\,000$<br>$C_2 = 11\,200\,000 + 11\,200\,000 \times 0.08 + 400\,000 = 12\,496\,000$<br>$\frac{C_1}{C_0} \neq \frac{C_2}{C_1}$ and $C_1 - C_0 \neq C_2 - C_1$ | **0.25**<br>**0.25**<br>**0.25**<br>**0.25** |
| | b- | $C_{n+1} = C_n + 0.08C_n + 400\,000 = 1.08C_n + 400\,000$ | **0.5** |
| 2) | a- | $U_{n+1} = 1.08(C_n + 5\,000\,000) = 1.08\,U_n$; $(U_n)$ is a geometric sequence of common ratio $r = 1.08$ and of first term $U_0 = 15\,000\,000$. | **1** |
| | b- | $U_n = U_0 \times r^n = 15 \times 1\,000\,000 \times 1.08^n$ and $C_n = 15 \times 1\,000\,000 \times 1.08^n - 5\,000$ | **0.5**<br>**0.5** |

| | | | |
|---|---|---|---|
| | **c-** | $C_{18} = 15\,000\,000 \times 1.08^{18} - 5\,000 = 54\,940\,000$; the capital of a child whose parents accepted the offer of this bank, after 18 years, is 54 940 000 LL | **0.5** |

| | **Question IV** | **Mark** |
|---|---|---|
| A)1 | h(1) = 2.8 then a + b = 2.8<br>h'(1) = -1.2 them b -1 = -1.2 therefore b = -0.2 and a = 3 | 1 |
| A) 2 | $lim_{x \to 0}\, h(x) = +\infty$ ; $lim_{x \to +\infty}\, h(x) = -\infty$<br>$\begin{array}{c\|lcr} x & 0 & & +\infty \\ \hline h'(x) & & - & \\ \hline h(x) & +\infty & \searrow & -\infty \end{array}$ | 1 |
| B)1 | $lim_{x \to +\infty}\, g(x) = 3.$ y = 3 is an asymptote of (C). | 0.5 |
| B)2 | g'(x) = g(x) = 0.6 $e^{-0.2x}$ but $x$ > 0. then, g is strictly increasing over ]0 ; +∞[.<br>g(0) = 0<br>$\begin{array}{c\|lcr} x & 0 & & +\infty \\ \hline g'(x) & & + & \\ \hline g(x) & 0 & \nearrow & 3 \end{array}$ | 1 |
| B)3 | Let k(x) = g(x) – h(x) = 3( 1– $e^{-0.2x}$) – (3 – 0.2x - lnx) = 0.2x + lnx – $e^{-0.2x}$<br>We have: k(2.93) × k(2.95) < 0 ,<br>then ($C_1$) cuts (C) at a point of abscissa α with 2.93<α<2.95 | 1 |
| B)4 |  | 1 |

| | | |
|---|---|---|
| C) 1 | p = 2, S(2) =  3 – 0.2(2) – ln 2 = 1.90685<br>the supply corresponding to a price of 2 000 LL is 1907 articles. | 0.5 |
| C) 2 | D(p) = 1.5,  or $3(1-e^{-0.2p}) = 1.5$  then $p = \frac{ln2}{0.2} = 3.47$<br>the price for a demand of 1500 items is 3470 L.L | 1 |
| C)3a | $\alpha$ = Equilibrium price = 1 000×2.94 = 2940 LL | 0.5 |
| C)3b | The total revenue = p × D(p) = 2940× 1330 = 3910200 L.L | 0.5 |