


المادة: الرياضيات الشهادة: المتوسطة نموذج رقم -١- المدة: ساعتان	الهيئة الأكاديمية المشتركة قسم: الرياضيات	 المركز التربوي للبحوث والإنماء
--	--	---

نموذج مسابقة (يراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

ارشادات عامة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الوارد في المسابقة.

I - (2 points)

Consider the three numbers A, B and C:

$$A = \frac{33 \times 10^{-4} \times 30 \times 10^2}{36 \times 10^{-2} \times 22 \times 10} \quad ; \quad B = \frac{7 - \frac{11}{3}}{1 - \frac{1}{6}} \quad ; \quad C = (\sqrt{2} - 1)^2 + (\sqrt{2} + 1)^2$$

All details of calculation must be shown.

- 1) Write A as a fraction in its simplest form.
- 2) Show that B is a natural number.
- 3) Verify that $C = B + 16A$.

II – (4 points)

The perimeter of a rectangle is 28cm. If the length is decreased by 10% and the width is increased by 20%, then the perimeter of this rectangle will be 28.8cm.

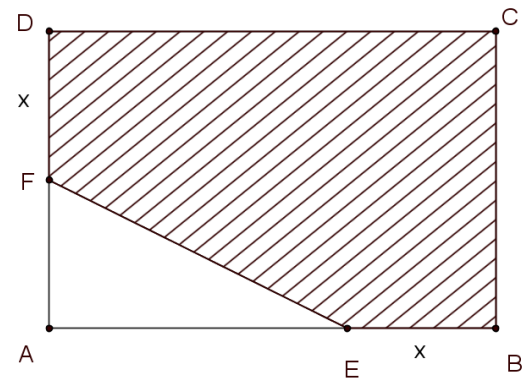
- a) Write a system of 2 equations of 2 unknowns to model the previous text.
- b) Verify that the original length is 8cm and calculate the original width.
- c) Determine the nature of quadrilateral resulting from modification of dimensions of the rectangle.

III – (4 points) in the figure at the right :

- x is a length expressed in cm such that $0 < x < 4$.
- ABCD is a rectangle such that $AB = 6\text{cm}$ and $AD = 4\text{cm}$.
- $BE = DF = x$

Denote by Y the area of the shaded part.

- 1) Prove that $Y = -\frac{1}{2}(x^2 - 10x - 24)$
- 2) a. Verify that $Y = -\frac{1}{2}((x - 5)^2 - 49)$.
b. Determine x so that $y = 20$.
- 3) Z is the area of a square with side $(x+2)$.
a. Express Z in terms of x .
b. Simplify $\frac{Y}{Z}$.
c. Can we calculate x if $Y = Z$?



IV - (5 points)

In an orthonormal system of axes $(x'Ox, y'Oy)$, consider the points $A(3; 0)$ and $B(-1; 2)$.

Let (d) be the line with equation $y = 2x + 4$.

- 1) a. Plot the points A and B.

- b. The line (d) intersects $x'Ox$ at E and $y'Oy$ at F. Calculate the coordinates of points E and F, then draw (d).
 - c. Verify that B is the midpoint of [EF].
- 2) a. Determine the equation of line (AB).
- b. Verify that (AB) is perpendicular bisector of [EF].
- 3) Consider the point $H(0 ; \frac{3}{2})$.
- a. Verify, that H is on the line (AB).
 - b. Show that H is the orthocenter of the triangle AEF.
- 4) Let (C) be the circle with diameter [AF] and (Δ) the line passing through A and parallel to (EH).
- a. Verify that O and B are on the circle (C).
 - b. Write an equation of the line (Δ).
 - c. Show that (Δ) is the tangent to (C).

V- (5 points)

In the adjacent figure at the right:

- $AB = 5$ cm.
- (C) is the circle with diameter [AB] and center O.
- E a point on (C) such that $AE = 3$ cm.
- The tangent to (C) at B intersect (AE) at F.

1) Copy the figure.

2) a. Calculate BE

b. Prove that the two triangles AEB and ABF are similar.

c. Deduce BF and EF.

3) L is a point on (FB) such that $BL = \frac{15}{4}$.

a. Compare $\frac{FE}{EA}$ and $\frac{FB}{BL}$.

b. Deduce that (BE) is parallel to (AL).

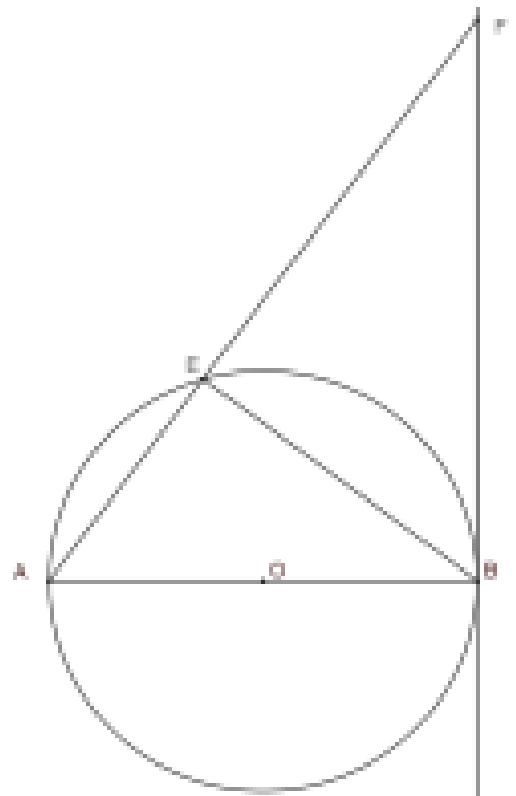
c. Show that $AL = \frac{25}{4}$


4) The line (EO) intersects the circle (C) at H. Let G the midpoint of [BL].

a. Prove that EAHB is a rectangle. Deduce that H is on (AL).

b. Prove that (GH) is tangent to (C).

c. Calculate, rounded to the nearest degree, the measure of \widehat{GHB} .



المادة: الرياضيات الشهادة: المتوسطة نموذج رقم - ١ - المدة : ساعتان	الهيئة الأكاديمية المشتركة قسم : الرياضيات	 المركز التربوي للبحوث والإنماء
---	---	---

أسس التصحيح (تراعي تعليق الدروس والتوصيف المعدل للعام الدراسي ٢٠١٦-٢٠١٧ وحتى صدور المناهج المطورة)

Question I		
	Answers	note
1	$A = \frac{33 \times 10^{-4} \times 30 \times 10^2}{36 \times 10^{-2} \times 22 \times 10} = \frac{9 \times 10^{-1}}{72 \times 10^{-1}} = \frac{1}{8} \quad \frac{1}{4} + \frac{1}{4}$ $B = \frac{\frac{10}{3}}{\frac{5}{6}} = 4, \quad \frac{1}{4} + \frac{1}{4}$ $C = (\sqrt{2} - 1)^2 + (\sqrt{2} + 1)^2 = 2 - 2\sqrt{2} + 1 + 2 + 2\sqrt{2} + 1 = 6 \quad \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$	1 ^{3/4}
2	$16A + B = 2 + 4 = 30$ $C = 6$, so $C = B + 16A$.	1/4
Question II		
a	$2x + 2y = 28 \text{ cm}$ $(1-0,1)x + (1+0.2)y = 28.8 \text{ cm}$	1 ^{1/4}
b	$x=8, y=6$	1
c	$1,2y=7.2$ et $0,9x=7.2$ Therefore the quadrilateral is a square.	3/4
Question III		
1	Area of hatched area $Y = 24 - \frac{(4-x)(6-x)}{2} = \frac{-x^2+10x+24}{2} = -\frac{1}{2}(x^2 - 10x - 24)$.	1
2.b	$20 = -\frac{1}{2}((x-5)^2 - 49)$ alors $(x-5)^2 - 49 = -40, (x-5)^2 = 9$ $x-5=3$ or $x-5=-3$ so $x=8$ (unacceptable) ou $x=2$. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$	1 ^{1/4}
3.a	$Z = (x+2)^2$	1/4
3.b	$\frac{Y}{Z} = \frac{-\frac{1}{2}(x-12)(x+2)}{(x+2)^2} = \frac{-\frac{1}{2}(x-12)}{(x+2)} = \frac{-(x-12)}{2(x+2)}$ (with $x \neq -2$)	1/2
3.c	$Y=Z$ so $\frac{-(x-12)}{2(x+2)}=1$ then $-(x-12)=2(x+2)$ so $x=\frac{8}{3}$ acceptable.	1

Question IV

1.a		$\frac{1}{2}$
1.b	E(-2;0) and F(0 ; 4)	$\frac{1}{2}$
1.c	$x_B = \frac{(x_E+x_F)}{2} \quad y_B = \frac{(y_E+y_F)}{2}$	$\frac{1}{2}$
2.a	<p>The equation of (AB) : $y = a x + b$</p> $a(AB) = \frac{(y_B - y_A)}{(x_B - x_A)} = \frac{-1}{2}$ <p>and $y_B = \frac{-1}{2} x_B + b$ so $b = \frac{3}{2}$.</p>	$\frac{3}{4}$
2.b	slope (AB) \times slope (d) = -1 and (AB) through B middle of [EF] so (AB) is the mediator of [EF].	$\frac{1}{2}$
3.a	$y_H = \frac{-1}{2} x_H + \frac{3}{2}$. so H is a point of (AB)	$\frac{1}{4}$
3.b	(FH) \perp at (EA) and (AB) \perp at (EF) , (AB) and (FH) meet in H then H is the orthocenter of the triangle AEF.	$\frac{3}{4}$
4.a	$\widehat{ABF} = 90^\circ$ (ABF triangle inscribed in a semicircle of diameter [AF]) $\widehat{AOF} = 90^\circ$ (AOF triangle inscribed in a semicircle of diameter [AF]) Therefore B and O are two points of the circle.	$\frac{1}{2}$
4.b	<p>The equation of (Δ) : $y = a x + b$</p> $a(\Delta) = a(EH) = \frac{(y_E - y_H)}{(x_E - x_H)} = \frac{3}{4}$ <p>and $y_A = \frac{3}{4} x_A + b$ so $b = \frac{9}{4}$.</p>	$\frac{3}{4}$
4.c	(EH) \perp at (FA) and (Δ)//at (EH) then (Δ) \perp at (FA) in A so (Δ) is tangential to the circle (C) in A.	$\frac{1}{2}$

5.b	<p>In the triangle BHL rectangle in H on $HG = GB = GL$ (the median is half the hypotenuse)</p> <p>Then the two triangles OBG and OHG are isometric.</p> <p>$\widehat{GHO} = \widehat{OBL} = 90$ then BH tangent to (C).</p>	$\frac{1}{2}$
5.c	<p>$\cos \widehat{GBH} = \frac{BH}{BL} = \frac{\frac{3}{4}}{\frac{15}{4}} = \frac{4}{5}$</p> <p>Alors $\widehat{GBH} = \cos^{-1}\left(\frac{4}{5}\right) = 36,8^\circ \approx 37^\circ$</p>	$\frac{1}{2}$