

Building up **MATHEMATICS**

Secondary Education
Third Year
Literature and Humanities Section



Educational Center for Research and Development



National
Textbook

New Curricula

SPECIMEN

Republic of Lebanon

Ministry of National Education, Youth and Sports

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Secondary Education
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*this manual is the Literature and Humanities version
of the manual of the Sociology and Economics section*

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The National Textbook Project

By issuing the textbooks for the third year of each educational cycle, the Educational Center for Research and Development will have completed the third and last installment of books called for by the New Curricula. We are placing these books in the hands of students with the great hope that we are moving, step by step, toward the goal of acquiring sound learning, using sophisticated educational means and up-to-date methodology that encourage and reinforce individual thinking and research, acquisition of skills, development of ethical and national attitudes, the feeling of national belonging as well as the feeling of belonging to humanity at large.

The on-going revolution in information, communication and educational-aids technology has undoubtedly limited the role of the textbook and lowered the rank it used so recently to occupy. However, in our society and in many other societies, the textbook remains the basic means of education, and it is our duty to exert our utmost effort and care to come up with the best product as to form and content. Yet we should not lose sight of the fact that the textbook is not sufficient by itself but should rather be used as a stepping stone to access other sources of information. What is important is to keep a clear vision and maintain the right course toward our objective. The means should not turn into the end and the student should always remain the focus of the learning/teaching process.

No one ignores or denies the fact that textbook writing requires very high academic and educational qualifications and very wide field experience. The authors committees undeniably possess such qualities. Yet the textbooks of the last two years contained some negative aspects. Such is the nature of human work, no matter how good the intentions or how great the effort extended. Here constructive criticism constitutes a real contribution to raising the standard of authorship, minimizing errors and filling gaps. We say that, with all due appreciation and respect to all those who have contributed to the success of this project.

The Educational Center for Research and Development is embarking this year on a process of evaluating the New Curricula and related textbooks, teacher training courses and student achievement. This is a natural and necessary step now that the new system has been put into effect. This process aims at identifying the curricular objectives that have been achieved as well as those that have not been achieved, with a view to proceeding with the positive aspects and correcting the negative ones.

As part of this correction process, we plan to review the versions that have been issued in order to secure good textbooks for our students, who always deserve the best.

March 13, 2000

**President, Educational Center for
Research and Development**

Nemer FRAYHA

PREFACE

The idea that Mathematics, Sciences, and Technology have close and profitable connections is widely acknowledged. It is known that economics use the Mathematics at a large scale, and that it was at the origin of what is known currently as Financial Mathematics. For a few decades, the use of Mathematics in the social sciences becomes an increasing extensive curve. These sciences contribute, in their turn, to the development of certain branches of Mathematics, for example, the role played by sociologists in the development of statistics.

Statistics, Probabilities, Financial Mathematics, Analysis, Graphical interpretation, Equations and Logic are the titles of this handbook. These various topics expose the study of Mathematics rigorously, but stressing at the same time the practical and useful aspect of the exposed methods. Our objective is to make the student acquire the mathematical needs so as to continue the studies in various fields, for example, the fields of philosophy, law, history, psychology, communication etc..., and also to practice a number of careers in administration, banks, education, craft industries and other various companies. Our major concern is to present this basic knowledge and to relate it to the daily life problems, hoping that students feel the important role the Mathematics play in the comprehension and analysis of the problems revealed in society, and in making the good decisions to solve these problems.

We have also tried to incorporate in each chapter complete review for the topics treated in former classes; we have proceeded thus to avoid, as much as possible, the use of other handbooks. Part of this review is mentioned in the course, which explains the great emphasis put on this issue, the other part is reviewed in exercises and problems, where the student requires total knowledge of the concerned topic.

We finally wish we have made the task of the student and the teacher as easy and pleasant as possible.

The authors.

HOW TO USE THIS MANUAL

I. Introduction

Gives a general idea of the theme and fixes the objectives.

1 INTRODUCTION TO MATHEMATICAL LOGIC

INTRODUCTION

Logic is the set of the rules, which manages the mental approach in the search of the truth. The roots of logic may be traced to Aristotle, who systemized and codified the subject in a way that was not significantly surpassed for over two millennia. Modern logic, however, stems largely from the work of the German philosopher Gottlob Frege in the late nineteenth century, and has developed tremendously during the twentieth century. Today logic has applications in many areas besides philosophy, including mathematics, linguistics, engineering, and computer science.

Among the various definitions of logic we will retain the one which summarizes best the contribution of this discipline to mathematics: "Logic is above all the discipline which treats correct inference" (L. Vax). The inference being the reasoning by deduction, an operation that consists to refer to one or many true statements, to reach one or many right statement(s). In other words, logic brings to mathematics rules for correct proofs.

However, sometimes our logical reasoning is faulty and errors can result. Therefore, it is important to identify the laws underlying logical derivations. Basic to logical derivations are propositions, which are statements that are either true or false. Propositions can be combined by various connectives to form compound propositions. There are certain propositions, called tautologies, that are always true. Tautologies give rise to logical implications and logical equivalences. Logical implications are basic to sound reasoning, and logical equivalences provide the means to manipulate propositions algebraically.

The objectives of this chapter are the following:

1. to identify a proposition;
2. to recognize and use the basic logical operations;
3. to use the truth table.

II. Activities

(Introduction to the subject)
Prepare the field to construct new concepts by using pre-known information.

ACTIVITIES

Activity 1. Only sentences

The philosophy teacher writes on the board the two following groups of sentences:

1 - "The man is an alive human being"
"Lebanon is in Europe"
"Number 5 is even"
"Joubran Khalil Joubran is a Lebanese philosopher"

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III. Course

(Panoramic idea)
Gathers up definitions and properties and indicates the main uses and applications.

TEXT

1 GENERAL INFORMATION ON PROPOSITIONS

Definition

Any statement that is either true or false is called a **proposition**.

To the question "is this statement true?" One must be able to answer without ambiguity by yes or no.

IV. FOCUS

(Notions to remember)
Summarizes the course, indicates the efficient methods and gives useful advice.

FOCUS

♦ A proposition is a statement that can say is either true or false ...

♦ To determine the truth value of the negation of a proposition or the composite of two or several propositions, we apply the following rules:

1. p is true if, and only if, p is false.
2. $p \wedge q$ is true if, and only if, p and q are both true.
3. $p \vee q$ is true if, and only if, at least one of the two propositions p or q is true.
4. $p \rightarrow q$ is true if, and only if, p is false or if p and q are both true. In that case one can say that p implies q , and one can write $p \Rightarrow q$ (explain the brackets).
5. $p \leftrightarrow q$ is true if, and only if, p and q are both true, or both false. In that case one can say that p and q are equivalent and we can write $p \Leftrightarrow q$ (indicate the brackets).

♦ A tautology is a proposition which is always true and a contradiction is a proposition which is always false.

♦ To show that a proposition is true, one often uses, beside the preceding rules, the following principles:

1. The deductive principle: $p \wedge (p \rightarrow q) \Rightarrow q$.
2. The syllogism rule: $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow p \rightarrow r$.
3. The reasoning by contradiction: p is true if $\neg p$ leads to a contradiction.
4. The contrapositive principle: to show that $p \rightarrow q$, we show that $\neg q \rightarrow \neg p$.

HOW TO USE THIS MANUAL

V. EXERCISES

(Preliminary steps)

Help to assimilate the results, to master the new techniques and to solve direct application problems.

EXERCISES

- 1 8 students obtained in an exam the following grades: 37, 65, 32, 49, 50, 69, 70, 62. Put these grades in increasing order and find the range of the data.
- 2 Fouad has paid, during the last six months, the following telephone bills: 35 500 LL, 33 600 LL, 55 000 LL, 29 300 LL, 45 300 LL. Find the range of this data.
- 3 The minimum temperatures registered during 8 consecutive days were: 20, 19, 18, 19, 20, 21, 22, 21. Find:
 - a) the sum of the deviations from the mean.
 - b) the absolute deviation from the mean.
- 4 The following given represents the waiting time (in minutes) of 10 clients in a bank: 10, 8, 7, 15, 14, 5, 10, 11, 9, and 11. Find:
 - a) the range;
 - b) the absolute deviation from the mean;
 - c) the variance;
 - d) the standard deviation;
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VI. Self evaluation

(An applicable test)

Allows the students to control by themselves their learning and their capacities to use the new results and techniques.

SELF-EVALUATION

- 1 Answer the following propositions by TRUE or FALSE:

	TRUE	FALSE
1. The range of a statistical data is the difference between the standard deviation and the average.		
2. Two different data sets can have the same average.		
3. Small standard deviation indicates that the values are widely dispersed.		
4. The sum of the deviations from the mean is null.		
5. We can calculate the absolute deviation from the mean, the variance and the standard deviation without using the average.		
6. A statistical data of equal modalities has a zero standard deviation.		

VII. Problems

(Analysis and syntheses)

Help to use the new notions simultaneously with the old ones to solve application problems taken from diversified quotidian situations.

PROBLEMS

- 1 We give the class centers and the relative frequencies of two different distributions in the following tables:

Data A		Data B	
Class center	Frequency	Class center	Frequency
18	43	2	1
23	36	5	6
28	25	8	11
33	14	11	23
38	8	14	16
43	16	17	7
48	29	20	4
53	33	23	1
58	41		

Calculate the average, the variance and the standard deviation of each of the two distributions.

VIII. Just for fun

(To go further)

Gives the possibility to think about notions which prolong the course and which are useful and pleasant to know.

JUST FOR FUN

Two teachers of a class of 20 students, exchange their impressions after an exam. The French teacher expresses his satisfaction. The math teacher did very well in class. French exams because the class average is 82.7% and the success rate is 85%. The math teacher expresses his worries. He says, most of my students have failed their math exam. The average is 52.2% and the success rate is only 45%. Knowing that the passing mark in each subject is any mark $\geq 10/20$, give two impressions of grades showing these two situations. Justify your answer by using the statistical techniques.

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