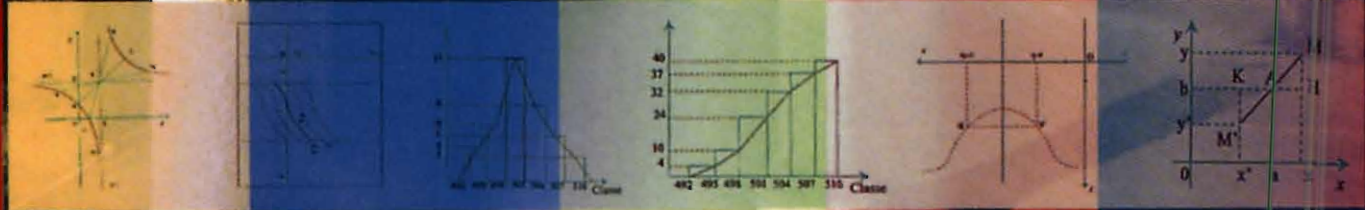
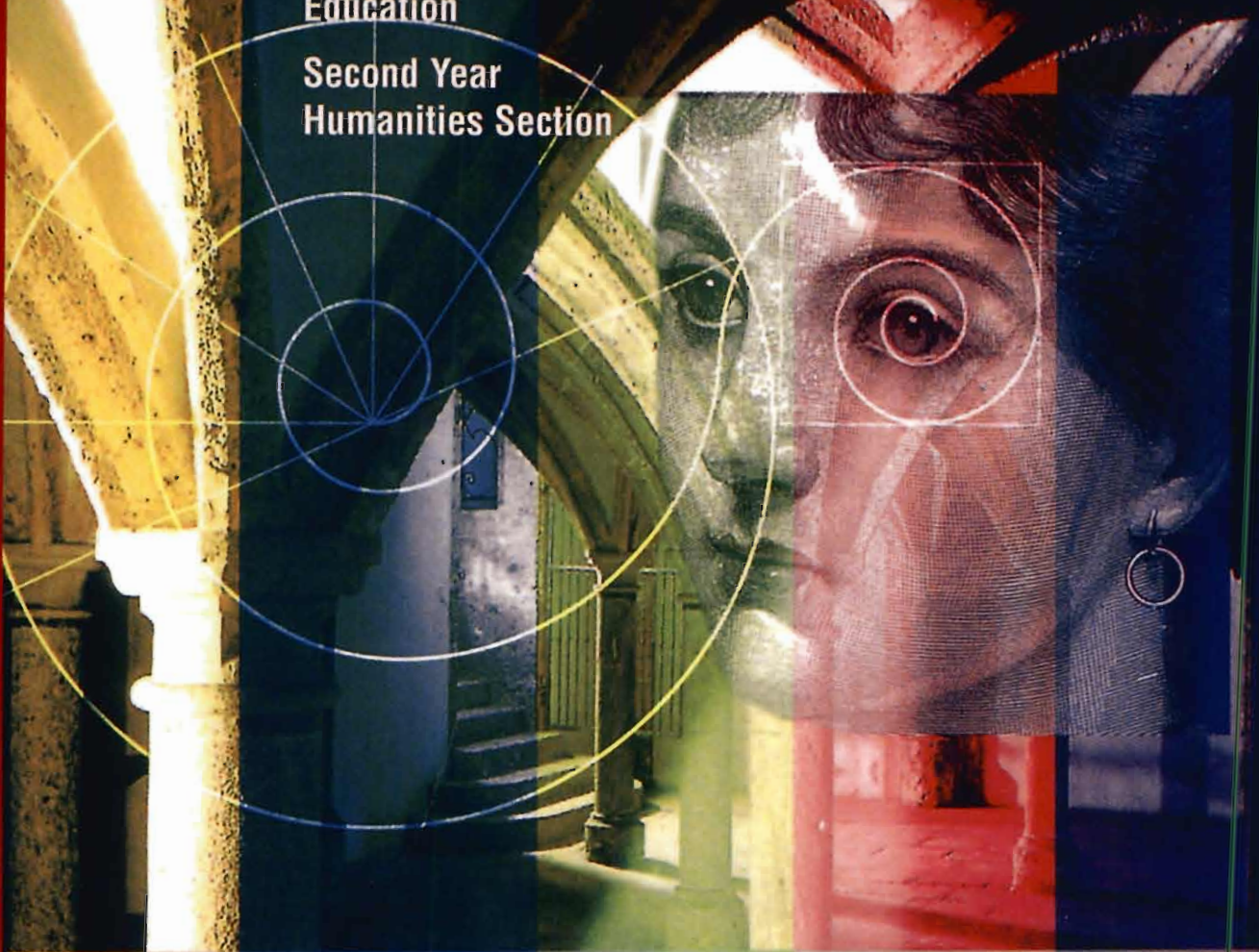


BUILDING UP MATHEMATICS

Secondary
Education

Second Year
Humanities Section



Republic of Lebanon

Ministry of National Education, Youth and Sports

SPECIMEN

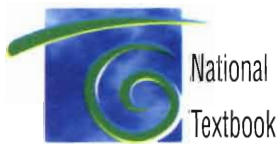
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Secondary Education

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Humanities Section

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BUILDING UP MATHEMATICS

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The National Textbook Project

This is the second installment of textbooks completed by the Center as part of a three-stage effort to produce the books called for by the New Curricula. We are placing these books in the hands of students with the great hope that we are moving, step by step, toward the goal of acquiring sound and modern learning, using sophisticated educational means and up-to-date methodology that encourage and reinforce individual thinking and research, the acquisition of skills, the development of ethical and national attitudes, the feeling of national belonging as well as the feeling of belonging to humanity at large.

The on-going revolution in information, communication and educational technology has undoubtedly limited the role of the textbook and lowered the rank it used so recently to occupy. However, in our society and in many other societies, the textbook remains the basic means of education, and it is our duty to exert our utmost effort and care to come up with the best product as to form and content. Yet we should not lose sight of the fact that the textbook is not sufficient by itself but should rather be used as a stepping stone to access other sources of information. What is important is to keep a clear vision and maintain the right course toward our objective. The means should not turn into the end and the student should always remain the focus of the learning/teaching process.

No one ignores or denies the fact that textbook writing requires very high academic and educational qualifications and very wide field experience. The authors committees undeniably possess such qualifications and qualities, yet last year's textbooks did contain some faults and gaps which were duly pointed out by researchers in many articles, and, indeed, we have benefited from some of them. Such is the nature of human work, no matter how good the intentions or how great the effort exerted.

Constructive criticism is a real contribution to raising the standard of authorship, minimizing errors and filling gaps. We only hope that criticism will always be objective and motivated by a desire to enhance educational reform in order to achieve better products.

A favorite adage handed down from our old scholars: "He who criticizes you is as helpful as a co-author". Let all criticism directed at the Center be of this caliber.

In closing, we hope that we all will have benefited from our experience and that the textbooks of the third and final stage be closer to realizing our hopes and more beneficial to our students. We are now preparing ourselves to assess the parts so far achieved of the new curricula and to assure that our educational movement is proceeding on the right track for achieving the best results.

June 2, 1999

**President, National Center for Educational
Research and Development**

Nemer FRAYHA

PREFACE

This book is an appealing and user-friendly mathematics handbook aimed at students in the second year of secondary education in the humanities section.

We set out on this project with the conviction that mathematics is, nowadays, essential to everyone in his or her social and professional lives. The wide range of activities demonstrates, on one hand, the diversity of knowledge required and on the other the methodology to acquire it. The topics of the official curriculum are addressed to meet the needs of the students. Instead of theoretical study of the classical notions of mathematics such as equations, derivatives, primitives, graphs, and so on, we have adopted a practical approach stressing on the application of these notions in specific areas particularly socioeconomic.

Clearly laid out probabilities are the first step of this new approach. They are taught as simply as possible as this area of mathematics, along with statistics, plays an increasingly important role in today's economic and social lives.

In this book, we aim to present mathematics following methodologies designed for it and adapted to it. We hope to motivate the students and to encourage them to find their own way in the world of mathematical notions and concepts so they can discover and benefit from their own mastery of these essential life skills.

The authors

USING THIS BOOK

I. INTRODUCTION

Before executing a job, we should have a clear idea about it. We have to define and specify the objectives.

1 ORDER RELATIONS

INTRODUCTION

In a given set, there are relations that connect some of its elements. For example, among the elements of \mathbb{N} , there are relations relative to the size (or magnitude), to the parity (evenness of numbers, as opposed to oddness), to the divisibility, etc. Each one of these relations satisfies a certain number of properties. In a given set, when a relation satisfies three specified properties, which will be defined later in this chapter, then the relation is called an order relation. In this manner, we can compare two points M and M' belonging to an axis of origin O using their abscissas OM and OM' .

In this chapter, we will see that the properties of the natural order axis a set of ordered points, relations in \mathbb{R} are shared by other relations. In other sets whose elements are not necessarily numerical numbers. Thus we obtain ordered sets other than the subsets of \mathbb{R} .

The objectives of this chapter are:

- To define the binary relations in a set.
- To define and to study the order relations.

Prerequisites

- Elementary arithmetic and algebra.
- Cartesian plane.

II. ACTIVITIES

A field where we have to use acquired knowledge, strengthen the capacity to acquire more, and prepare for additional work.

ACTIVITIES

Activity 1. In the, there are negligible quantities

(1) Use the figure to the right to calculate the area S_1 and S_2 of the square $OACD$, respectively.

(2) Calculate the function $f: \mathbb{R} \rightarrow \mathbb{R}$ in terms of x .

(3) Use a geometric method to verify that $\sin(\theta + \alpha) = \sin\theta \cos\alpha + \cos\theta \sin\alpha$.

Activity 2. Find me actively than we answer

ABC is a triangle right at A , AM is an altitude.

(1) Write the corresponding angles between AM , BC , and CM .

(2) Let $BM = x$ and $CM = y$, complete the following table:

x	100	150	200	300
y				
$f(x)$				
$f(y)$				

* Find the limit of $f(x)$ as $x \rightarrow 0^+$.

* Find the limit of $f(y)$ as $y \rightarrow 0^+$.

* Find the limit of $f(x)$ as $x \rightarrow \infty$.

* Find the limit of $f(y)$ as $y \rightarrow \infty$.

* Find the limit of $f(x)$ as $x \rightarrow 0^+$.

* Find the limit of $f(y)$ as $y \rightarrow 0^+$.

* Find the limit of $f(x)$ as $x \rightarrow \infty$.

* Find the limit of $f(y)$ as $y \rightarrow \infty$.

III. TEXT

Where we find appropriate answers to solve problems. The fruitful relationship between the practice and the conclusions that we make help us become more efficient.

TEXT

Write the concept from Chapter 4

I Histogram of frequencies

This is a graphical representation of the data. It is a two-dimensional graph with the classes (disposed on the horizontal axis) and the frequency (disposed on the vertical axis) on the axes on the left. The height of each bar is equal to the frequency of the class in question. Note: An interval has not been determined (division of a class into the lower class and the next higher class). The graph is called a histogram.

II Histogram of the increasing cumulative frequencies

We increasing cumulative frequency when we give the number of observations up to and including the class. To draw the graph, the limit of the class are disposed on the horizontal axis for each interval in value x_i . We construct a rectangle whose height is equal to the increasing cumulative frequency of the class.

III Histogram of the decreasing cumulative frequencies

A decreasing cumulative frequency column of a class gives the number of observations starting from and including this class. To draw the graph, the limit of the class are disposed on the horizontal axis for each interval in value x_i . We construct a rectangle whose height is equal to the cumulative decreasing frequency of the class.

We can construct the histogram of increasing and decreasing cumulative relative frequencies. The values are disposed on the horizontal axis and the increasing or decreasing cumulative relative frequency on the vertical axis.

IV. FOCUS

A summary of the procedure and the tools necessary to accomplish the work.

FOCUS

f is a differentiable function on an interval I , f' is the derivative of f on I , then:

- 1) the sign of $f'(x)$ shows the variation of f on I ;
- 2) the changes of sign of $f'(x)$ produce the local extrema of f in I ;
- 3) for $a \in I$, $f'(a)$ is the slope of the tangent, at the point $A(a, f(a))$, to the curve C of f ;
- 4) if $f'(a) = 0$ for $a \in I$, the tangent at A is parallel to the x -axis.

* If $a \in I$ and if $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \pm \infty$ then the tangent at A to C is vertical.

V. EXERCISES

Everything is incorporated to ensure success.

EXERCISES

1. Solve graphically each of the following systems:

a) $\begin{cases} 2x + y = 5 \\ x - y = 1 \end{cases}$

b) $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

c) $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

d) $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

e) $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

f) $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$

2. Draw the following systems as completed by a graph, showing the corresponding feasible region. Determine the coordinates of all vertex numbers of each region.

a) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

b) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

c) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

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e) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

f) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

3. Draw the following feasible regions, or find all x values for maximum or the minimum of the geometrical function.

a) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

b) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

c) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

d) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

e) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

f) $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

4. Find the optimal solution for the following problems:

a) Maximize $z = 2x + 3y$ subject to $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

b) Maximize $z = 2x + 3y$ subject to $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

c) Maximize $z = 2x + 3y$ subject to $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

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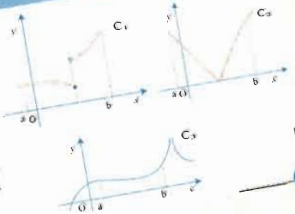
f) Maximize $z = 2x + 3y$ subject to $\begin{cases} x + y \leq 3 \\ x - y \leq 1 \end{cases}$

VI. SELF-EVALUATION

It is often necessary to take decisions alone and assume responsibility. At school and at work, one needs to face the truth and reality.

SELF-EVALUATION

1. The function f is defined by $f(x) = |x - 1|$. Is f continuous at $x = 1$?
2. f is defined by $f(x) = \frac{1}{x - 1}$. Can we say that f is continuous at $x = 1$?
3. Which of the curves to the right are those of continuous functions on $[a, b]$?



VII. PROBLEMS

To solve problems, we should rely on our analytical and synthesizing capacities. Furthermore, we should use different tools and strategies to succeed.

PROBLEMS

1. Consider the frequency distribution that follows:
- a) Calculate the relative frequency and the absolute frequency of each class.
- b) Draw the histogram and the polygon of frequency.
- c) How many data are there?
- d) How many data are there?
- e) How many data are there?
- f) How many data are there?

Class	Frequency
27-30	45
31-34	50
35-38	60
39-42	40
43-46	30
47-50	20
51-54	10
55-58	5
59-62	2
63-66	1

2. The following are the grades of 100 students in a math test.

78	79	80	81
82	83	84	85
86	87	88	89
90	91	92	93
94	95	96	97
98	99	100	101
102	103	104	105
106	107	108	109
110	111	112	113
114	115	116	117
118	119	120	121
122	123	124	125
128	129	130	131
134	135	136	137
140	141	142	143
146	147	148	149
150	151	152	153
156	157	158	159
160	161	162	163
166	167	168	169
170	171	172	173
176	177	178	179
180	181	182	183
186	187	188	189
190	191	192	193
196	197	198	199
200	201	202	203
206	207	208	209
210	211	212	213
214	215	216	217
218	219	220	221
222	223	224	225
228	229	230	231
234	235	236	237
240	241	242	243
246	247	248	249
250	251	252	253
256	257	258	259
260	261	262	263
266	267	268	269
270	271	272	273
276	277	278	279
280	281	282	283
286	287	288	289
290	291	292	293
296	297	298	299
300	301	302	303

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110	111	112	113
114	115	116	117
118	119	120	121
122	123	124	125
128	129	130	131
134	135	136	137
140	141	142	143
146	147	148	149
150	151	152	153
156	157	158	159
160	161	162	163
166	167	168	169
170	171	172	173
176	177	178	179
180	181	182	183
186	187	188	189
190	191	192	193
196	197	198	199
200	201	202	203
206	207	208	209
210	211	212	213
214	215	216	217
218	219	220	221
222	223	224	225
228	229	230	231
234	235	236	237
240	241	242	243
246	247	248	249
250	251	252	253
256	257	258	259
260	261	262	263
266	267	268	269
270	271	272	273
276	277	278	279
280	281	282	283
286	287	288	289
290	291	292	293
296	297	298	299
300	301	302	303

4. Group the "like like terms" among the ones below and simplify.

5. Which class has the maximum number of students?

6. How many students are there in total?

7. What is the percentage of students who passed the test?

8. How many students are there in total?

9. How many students are there in total?

10. How many students are there in total?

VIII. JUST FOR FUN

There are subjects that are attractive to some people and not to others. Our objective is to make mathematics enthralling, interesting, and fascinating.

JUST FOR FUN

On the given figures, C represents the graph of the trigonometric function $f(x) = \sin(x)$. C' is that of the cosine function $f(x) = \cos(x)$. C'' is that of the tangent function $f(x) = \tan(x)$. C''' is that of the cotangent function $f(x) = \cot(x)$. C'''' is that of the secant function $f(x) = \sec(x)$. C''''' is that of the cosecant function $f(x) = \csc(x)$. Which of the curves represents the function $f(x) = \sin(x)$?